MALLA REDDY ENGINEERING COLLEGE (AUTONOMOUS)
(Affiliated to Jawaharlal Nehru Technological University Hyderabad)
Maisammaguda, Dhulapally, (Post via Kompally), Secunderabad-500 100.

A
LECTURE NOTES

ON

POWER SYSTEM OPERATION AND CONTROL

2019 – 2020

III B. Tech II Semester

Mrs. K.AnithaReddy, Assistant Professor

SYALLUBUS

Code: 70218	POWER SYSTEM OPERATION AND CONTROL	L	T	P
Credits: 3		2	2	-

Prerequisites: Power System Generation and Distribution, Power System Analysis.

Course Objectives: This course deals with Economic operation of Power Systems, Hydrothermal scheduling and modelling of governors, turbines and generators. It emphasizes on single area and two area load frequency control and reactive power control.

MODULE I: Economic Operation of Power Systems 13 Periods

Optimal operation of Generators in Thermal Power Stations - Heat rate Curve – Cost Curve – Incremental fuel and Production costs - Input - Output characteristics - Optimum generation allocation with line losses neglected.

Optimum generation allocation including the effect of transmission line losses – Loss Coefficients - General transmission line loss formula.

MODULE II: Hydrothermal Scheduling 13 Periods

Optimal scheduling of Hydrothermal System: Hydroelectric power plant models, scheduling problems - Short term hydrothermal scheduling problem.

MODULE III: Load Frequency Control – I 13 Periods

A: Modeling of Governor, Turbine and Generators with corresponding block diagram representation and transfer function.

B: Single Area Load Frequency Control: Necessity of keeping frequency constant. Definitions of control area – Single area control – Block diagram representation of an isolated power system – Steady state analysis – Dynamic response – Uncontrolled case.

MODULE IV: Load Frequency Control – II 13 Periods

Proportional plus Integral control of single area and its block diagram representation - Steady state response – Load Frequency Control and economic dispatch control.

Load frequency control of two area system – Uncontrolled case and controlled case – Tie - Line bias control.

MODULE V: Reactive Power Control 12 Periods

Overview of Reactive Power control – Reactive Power compensation in transmission systems – Advantages and disadvantages of different types of compensating equipment for transmission systems. Load compensation – Specifications of load compensator. Uncompensated and compensated transmission lines: Shunt and Series Compensation (qualitative treatment).

TEXT BOOKS

1. Abhijit Chakrabarthi and Sunita Halder, **"Power System Analysis Operation and Control"**, PHI Learning Pvt. Ltd., 3rd Edition, 2010.
2. I.J.Nagrath and D.P.Kothari, **"Modern Power System Analysis"**, Tata McGraw Hill Publishing Company Ltd, 4th Edition, 2011.

REFERENCES

1. C.L.Wadhwa, **"Electrical Power Systems"**, New Age International (P) Limited, Pzaublishers, 4th Edition, 2005.

2. T.J.E. Miller, “**Reactive Power Control in Electric Systems”**, John Wiley & Sons, New York, 1982.
3. J.Duncan Glover, M.S.Sarma and Thomas J.Overbye, “**Power System Analysis and Design”,** Global Engineering Publisher, 5th Edition, 2012.
4. O.I.Elgerd, **“Electric Energy Systems Theory”**, Tata McGraw - Hill Education, 2nd Edition, 2003.
5. John J Grainger, William D Stevenson Jr, **“Power System Analysis”**, Tata McGraw – Hill Education, 2003.

COURSE OUTCOME

CO1:Determine the Scheduling of Thermal PowerPlant and Cost Optimization.
CO2:Computing the Optimal Scheduling of Hydro Thermal System.
CO3:Analyze the Steady state Response and Dynamic response of Single area Control.
CO4:To calculate the steady state response of load frequency control of two area system.
CO5:Analyze the different Reactive Power Compensator like Static Var Compensators,Synoronouscondensors,Shunt and series compensators to control the Voltage

LESSON PLAN

Theory Class: 14						
Module – I	**Economic Operation of Power Systems**			***Target Hours 14***		
Sl. No	**Date**	**Period Reqd.**	**Topics to be Covered**	**Ref. Book.**	**Date of completion**	**Remarks**
1	2.12.19	1	Optimal operation of Generators in Thermal Power Stations	T1 & T3		
2	4.12.19	1	heat rate Curve – Cost Curve –	T1 & T3		
3	5.12.19	2	Incremental fuel and Production costs, input-output characteristics, Optimum generation allocation with line losses neglected.	T1 & T2		
4	10.12.19	4	Optimum generation allocation including the effect of transmission line losses, – Loss Coefficients,	T1 & T2		
5	16.12.19	4	General transmission line loss formula.	T1 & T2		
6	21.12.19	2	Problems	T1 & T2		

Theory Class: 10						
Module II	**Hydrothermal Scheduling**			***Target Hours 10***		
Sl. No	**Date**	**Period Reqd.**	**Topics to be Covered**	**Ref. Book.**	**Actual Date of completion**	**Remarks**
1	23.12.19	2	Optimal scheduling of Hydrothermal System:	T1 & T2		
2	26.12.19	2	Hydroelectric power plant models	T1 & T2		
3	30.12.19	1	scheduling problem	T1 & T2		
4	2.01.20	3	Short term hydrothermal scheduling problem	T1 & T2		
5	6.01.20	2	Problems	T1 & T2		

Theory Class: 14						
Module - III	**Load Frequency control-I**			***Target Hours 14***		
Sl. No	**Date**	**Period Reqd.**	**Topics to be Covered**	**Ref. Book.**	**Date of completion**	**Remarks**
1	8.01.20	3	Modeling of Governor,turbine and Generator	T1 & T2		
2	13.01.20	4	transfer function. Single Area Load Frequency Control: Necessity of keeping frequency constant. Definitions of Control area – Single area control	T1 & T2		
3	17.01.20	4	Block diagram representation of an isolated power system – Steady state analysis	T1 & T2		
4	21.01.20	3	Dynamic response – Uncontrolled case.	T1 & T2		

Theory Class:12						
Module - IV		**Load Frequency Control II**		***Target Hours 12***		
Sl. No	**Date**	**Period Reqd.**	**Topics to be Covered**	**Ref. Book.**	**Date of completion**	**Remarks**
1	23.01.20	1	Proportional plus Integral control of single area and its block diagram representation, steady state response	T1 & T2		
2	3.2.20	2	Load Frequency Control and Economic dispatch control	T1 & T2		
3	5.02.20	2	Load frequency control of 2-area system	T1 & T2		
4	8.03.20.	2	uncontrolled case and controlled case, tie-line bias control	T3		
5	13.03.20	2	Problems	T1,T2&T3		

Theory Class: 12						
Module - V		**Reactive Power Control**		***Target Hours 12***		
Sl. No	**Date**	**Period Reqd.**	**Topics to be Covered**	**Ref. Book**	**Date of completion**	**Remarks**
1	13.03.20	2	Overview of Reactive Power control – Reactive Power compensation in transmission systems	T3 & T2		
2	15.03.20	3	advantages and disadvantages of different types of compensating equipment for transmission systems;	T3& T2		
3	16.03.20	2	load compensation – Specifications of load compensator	T3 & T2		
4	22.03.20 23.03.20	3	Uncompensated and compensated transmission lines	T3 & T2		
5	27.03.20 29.03.20	2	shunt and Series Compensation (qualitative treatment).	T3 & T2		

FACULTY IN-CHARGE **HOD/EEE**

Mapping Between Cos and Pos

	Po1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1	3	3	3	3	3							3
CO2	3	2	2	2								
CO3	3	2										
CO4	2	2		2	2							
CO5	3	3		3	3							3

MODULE I: Economic Operation of Power Systems

ECONOMIC OPERATION OF POWER SYSTEM

Introduction

The problem of Economic Operation of Power System or Optimal Power Flow (OPF) can be stated as:

"Allocation of the load (MW) amongst the various units of generating station and amongst the various generating stations in such a way that, the overall cost of generation for the given load demand is minimum".

Basically this is an optimization problem, the objective is to minimize the generation cost function subject to the satisfaction of given set of equality and inequality constraints. This problem is analyzed, solved and then implemented under the online condition of power system. The input data for the problem is the result of conventional power flow study. For a given load demand , the power flow study performs the calculation of active and reactive power generations, line flows, losses etc. Also the study furnishes some of the parameters for control purpose such as magnitude of voltage and the voltage phase differences. The economic scheduling problem can be understood as the solution of multiple power flow studies, where a particular power flow study results are more appropriate in the sense of cost of generation. As said before, the solution can not be optimal unless otherwise all the constraints of the system are not satisfied. we understand the economic scheduling problem in the coming sections, but first we consider the constraints of the problem.

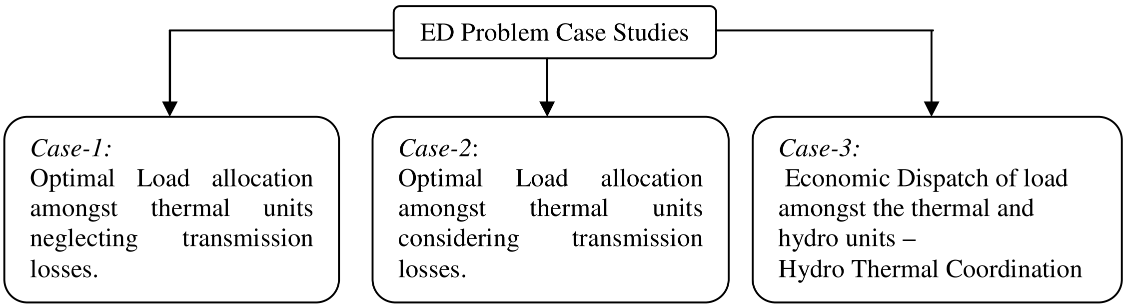
The problem of Economic Operation of system involves with two sub-problems. Namely, Unit Commitment (UC) and Economic Dispatch (ED). Basically UC problem is an off-line where as the ED is an on-line problem. The result of UC is the initial solution for ED problem.

Unit Commitment Problem

Generally the load curve of a plant which gives variation of load vs. time in a season follows a particular trend or pattern. Hence with a minimum degree of error, these curves can be predicted or can be forecasted. Let *N* number of generating units are available in the plant say. The problem of UC is to how the forecasted plant load should be allocated amongst the *N* units in such a way that the overall cost of generation of the plant is minimum. In other words, out of *N* units available, the solution of UC is a schedule of the units those must be kept 'on' (committed) condition or in 'off' (de-committed) condition. The problem of UC is more complex owing to the requirement for the satisfaction of large number of constraints like start-up and shut-down costs and times, fuel cost per KWh, minimum and maximum generating capacities etc.

Economic Dispatch Problem

In contrast to UC problem, ED assumes *N* number of units in all the different generating stations are already committed or in 'on' condition, the problem is to how the forecasted load should be shared amongst them in such a way overall cost of generation is minimum. The UC solution involves ED as a sub problem. That is, once the decision of units those should be committed is taken, next ED resolves the distribution of load amongst the committed units. If all the units are in 'on' condition, the UC does not exists and we have to take up only ED problem. This text book Economic Dispatch problem for the following case studies.



This chapter deals Case-1 study where as the next two chapters present the other case studies.

CHARACTERISTICS OF THERMAL UNITS

The cost of power generation by any thermal unit depends upon its characteristics. Performance of any unit can be understood by studying the generator's input-output characteristics and the cost curve. Characteristics of individual machines which are drawn in graphs are later converted into suitable mathematical equations using curve fitting methods. These mathematical equations are used for economic scheduling problem. The input of a thermal unit is kcal/hr in terms of heat supplied or Rs/hr in terms of cost of fuel and the output is in terms of MW.

Input-output or Heat Characteristic

Heat curve is graph drawn between fuel input in Btu/h or Kcal/h versus power output in MW on x-axis and y-axis respectively. Typical Curve for a thermal unit takes the form as shown in the Fig10.1.

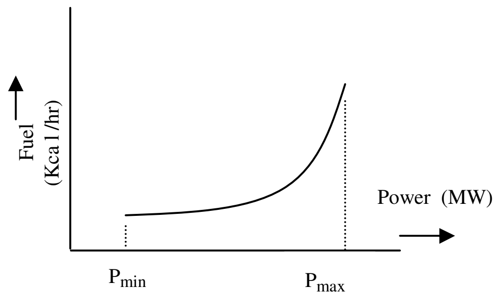


Fig 10.1 Heat Curve of a Thermal Unit

The slope of the curve at different powers can be calculated. The slope is defined as *heat rate* or *incremental fuel rate* which is the ratio of fuel input to the corresponding power output and has the
units Btu/MWh or Kcal/MWh.

$$\text{Incremental fuel rate} = \frac{dF}{dP} = \frac{\Delta\text{input}}{\Delta\text{output}}$$

Heat rate (slope of heat curve) can be calculated at every point in the heat curve and another characteristic Incremental heat curve can be obtained. This graph is drawn between Incremental fuel rate in Kcal/MWh versus Output in MW. The graph is shown in the Fig.10.2.

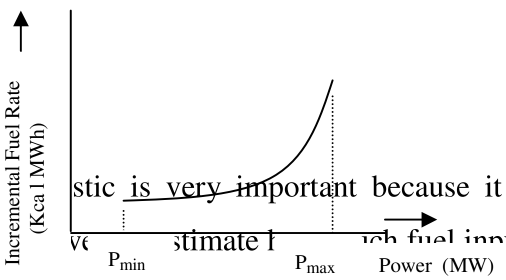


Fig 10.2 Incremental Fuel Rate Characteristic of a Thermal Unit

Incremental Fuel Ch[illegible]stic is very important because it gives thermal efficiency of the thermal unit through [illegible] ve [illegible] stimate [illegible] fuel input is required for every additional MW generation. Different units have Incremental Fuel rates differently for a given MW value. From the optimal [illegible] ate the load to the unit with minimum Incremental Fuel Rate.

Cost Curve and Incremental Fuel Cost Characteristic

Knowing the specific heat (in Kcal/Kg) of the coal used, fuel input (in Kcal/hr) and Incremental Fuel rate (in Kcal/MWh) can be converted into Kg/hr and Kg/MWh respectively. Again cost of each Kg of coal is known, Kg/hr and Kg/MWh can then be converted into Rs/hr and Rs/MWh.

Now, the incremental Fuel Cost curve can be drawn by taking incremental fuel cost in Rs/MWh as input on x-axis and power in MW as output on y-axis. Cost curve is shown in Fig 10.3.

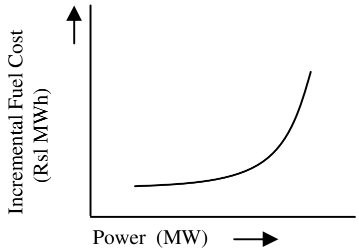


Fig 10.3 Cost Curve

Similarly the Input-output characteristic which is drawn in Kcal/hr versus MW can be converted into cost curve drawn Rs/hr versus MW as shown in Fig.10.4.

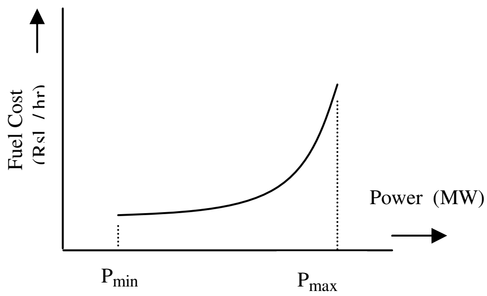


Fig 10.4 Cost Curve of a Thermal Unit

Let the fuel cost for i^{th} generator can be in the form as shown in Fig.10.4. The curve can be fitted in quadratic equation as:

$$C_i = \alpha_i + \beta_i P_i + \gamma_i P_i^2 \text{ Rs/hr.}$$

(10.7)

and Incremental fuel cost can be written in the form:

$$\frac{dC_i}{dP_i} = 2\gamma_i P_i + \beta_i \quad \text{Rs/MWh}$$

(10.8)

OBJECTIVE FUNCTION AND CONSTRAINTS FOR ECONOMIC DISPATCH PROBLEM

As discussed before ED problem deals with optimal allocation of total load demand P_D amongst *n*- number of units. The objective function for ED problem is presented below.

The problem is to minimize the function Eq.(10.9) subject to the satisfaction of constraints.

$$C_{Total} = f(P_{G1}, P_{G2}, \ldots, P_{Gn})$$

(10.9)

Where C_{Total} is the cost of Generation and $P_{G1}, P_{G2}, \ldots, P_{Gn}$ are individual generations of *n*-number of units

The ED problem may be treated as the parameter (cost) optimization subject to the satisfaction of system constraints. System constraints are of two types:

i) Equality constraints and
ii) Inequality constraints.

Equality constraints

These constraints are due to functional dependencies. Let f and g have functional dependencies and the problem is to minimize the function $f(x_1, x_2, x_3)$ say. The computed values of x_1, x_2, and x_3, be said as optimal, when they both minimize the f and satisfy the following equality constraints which are described by a set of equations :

$g_i(x_1, x_2, x_3) = 0$ for $i=1,2,3,\ldots\ldots\ldots x$ set of equations.

In the present ED problem, the equality constraint functional dependency equations are the static power flow equations. For the optimized scheduling of generation for the given demand, there is a necessity that the static power flow equations also need to be satisfied. The other equality constraint is the sum of power generation by individual units must be equal to total power demand as:

$$\sum_{i=1}^{n} P_{Gi} = P_D \quad for\ i = 1,2,\ldots\ldots.n.$$

(10.10)

or

$$g_i(P_{G1}, P_{G2}, \ldots.., P_{Gn}) = \sum_{i=1}^{n} P_{Gi} - P_D = 0$$

(10.11)

Inequality constraints

The following inequality constraints may be included in the ED problem.

1)Each Generating unit on *n*- number must operate below its maximum limit and above minimum limit.

$$P_i^{\min} \le P_i \le P_i^{\max} \qquad i = 1,2,\ldots,n.$$

2)Reactive power(Q) limits of individual units: Q value should be less than maximum value to avoid rotor over heating and should be more than minimum value to ensure proper power transfer capability of system.

3)Total MVA rating of individual units, to ensure there is no excess stator heating.

4)Over loading condition of equipment like transformers, lines etc.

5)Limits to voltage magnitudes and angles to ensure system has better stability margins. Low voltage problems give rise voltage instability and unsatisfactory operation of loads dominated by Induction Motor type. High voltage problems shall lead to insulation breakdowns.

6)Transformer tap positions: Tap position should be in between the minimum and maximum values.

7)Spare of reserve capacities of generating units: To meet sudden increase of power demand and maintain good steady state stability of system.

Above constraints basically are control parameters, which are required to be with in their limits for the satisfactory operation of Power system. Though the primary interest is the minimization of objective function, owing to operational limitations these constraints are important and hence must be included in the study.

LANGRANGE MULTIPLIER METHOD - OVERVIEW

The nonlinear function optimization problems such as ED can be solved by *Lagrange multiplier* method.

Nonlinear Function Optimization considering Equality Constraints

Let the problem is to minimize the function

$f(x_1, x_2, \ldots, x_n)$

subject to k number of equality constraints

$g_i(x_1, x_2, \ldots, x_n) = 0$ for $i = 1, 2, \ldots, k$.

The constrained function f can be written as unconstrained function with the help of Lagrange method as:

$$\pounds = f + \sum_{i=1}^{k} \lambda_i g_i$$

(10.12)

In Eq.(10.12), $\pounds$ is the Lagrange function and λ is the Lagrange multiplier. The necessary condition for minimum $\pounds$ can be obtained from:

$$\frac{\partial \pounds}{\partial x_i} = \frac{\partial f}{\partial x_i} + \sum_{i=1}^{k} \lambda_i g_i = 0 \quad \text{for } i=1,2,\ldots,n$$

(10.13)

$$\frac{\partial \pounds}{\partial \lambda_i} = g_i = 0 \quad \text{for } i=1,2,\ldots,k$$

(10.14)

Eq.(10.14) represent the original constraints. This Method is adopted in the ED problem and is explained in detail is the coming sections.

Nonlinear Function Optimization considering Equality and Inequality Constraints

Majority of Optimization problems contain both equality and inequality constraints. Let the problem is to minimize the function:

$f(x_1, x_2, \ldots, x_n)$

subject to k number of equality constraints

$g_i(x_1, x_2, \ldots, x_n) = 0$ for $i = 1, 2, \ldots, k$.

and m number of inequality constraints

$q_i(x_1, x_2, \ldots, x_n) \leq 0$ for $i = 1, 2, \ldots, m$.

The inequality constraints as mentioned earlier, are independent control parameters or undetermined quantities. These constraints are bounded to certain limits. By introducing m vector of μ undetermined quantities, the constrained function f can be written as unconstrained function with the help of Lagrange method as:

$$£ = f + \sum_{i=1}^{k} \lambda_i g_i + \sum_{j=1}^{m} \mu_j q_j$$

(10.15)

The necessary condition for minimum £ can be obtained from:

$$\frac{\partial £}{\partial x_i} = \frac{\partial f}{\partial x_i} + \sum_{i=1}^{k} \lambda_i g_i = 0 \qquad \text{for} \qquad i=1,2,\ldots,n$$

(10.16)

$$\frac{\partial £}{\partial \lambda_i} = g_i = 0 \qquad \text{for} \qquad i=1,2,\ldots,k$$

(10.17)

$$\frac{\partial £}{\partial \mu_j} = u_j \le 0 \qquad \text{for} \qquad j=1,2,\ldots,m$$

(10.18)

$$\mu_j u_j=0 \qquad \text{and} \qquad \mu_j>0 \qquad \text{for} \qquad j=1,2,\ldots,m$$

(10.19)

Note that Eq.(7.17) is original equality constraints. Above necessary conditions are known as *Kuhn-Tucker* conditions.

ECONOMIC DISPATCH PROBLEM –NEGLECTING TRANSMISSION LINE LOSSES

The problem of ED can be easily solved when the transmission line losses are neglected. As the losses are neglected, system model can be understood as shown in the Fig.10.5. In the figure, n number of generating units are connected to a common bus bar, collectively meeting the total power demand P_D. It should be understood that for any share of power demand by the units, will not involve losses.

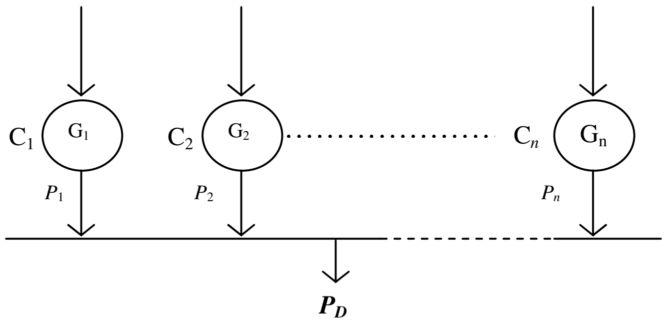


Fig 10.5 System with *n*- generators

Since transmission losses are neglected, total demand P_D is the sum of all generations of n-number of units. For each unit a cost functions C_i is assumed and the sum of all the costs computed from these cost functions of the units give total cost of production C_T.

$$C_T = \sum_{i=1}^{n} C_i \qquad \text{For } i=1,2,\ldots\ldots n$$

(10.20)

Where the cost function of i^{th} unit from Eq.(10.7) is:

$$C_i = \alpha_i + \beta_i P_i + \gamma_i P_i^2 \qquad (10.7)$$

Now, the economic dispatch problem is to minimize C_T, Subject to the satisfaction of the following equality and inequality constraints.

Equality constraint **:**

The total power generation by all the generating units must be equal to power demand.

$$\sum_{i=1}^{n} P_i = P_D$$

(10.21)

Where P_i= power generated by i^{th} unit

P_D= total power demand.

Inequality constraint :

Each generator can generate power with in the limits imposed.

$$P_i^{\min} \le P_i \le P_i^{\max} \qquad i = 1,2,\ldots,n.$$

(10.22)

Economic Dispatch problem can be carried out by excluding or including generator power limits i.e the inequality constraint.

SOLUTION OF ECONOMIC DISPATCH PROBLEM- NEGLECTING LOSSES & GENERATOR LIMITS

The constrained total cost function Eq.(10.20) can converted into unconstrained function by using Lagrange Multipliers as:

$$\pounds = C_T + \lambda(P_D - \sum_{i=1}^{n} P_i) \tag{10.23}$$

The conditions for minimization of Objective function can be found by equating partial differentials of the unconstrained function to zero as given below:

$$\frac{\partial \pounds}{\partial P_i} = \frac{\partial C_T}{\partial P_i} + \lambda(0-1) = 0$$

(10.24)

From the above equation,

$$\lambda = \frac{\partial C_T}{\partial P_i}$$

(10.25)

Since $C_i = C_1 + C_2 + \ldots\ldots. + C_n$

$$\frac{\partial C_T}{\partial P_i} = \frac{\partial C_i}{\partial P_i} = \lambda$$

(10.26)

From above equation the *co-ordinate equations* can be written as:

$$\frac{\partial C_1}{\partial P_1} = \frac{\partial C_2}{\partial P_2} = \frac{\partial C_3}{\partial P_3} \ldots\ldots\ldots = \frac{\partial C_n}{\partial P_n} = \lambda \tag{10.28}$$

Using Eq.(10.7),

$$\frac{\partial C_i}{\partial Pi} = \lambda = \beta_i + 2\,\gamma_i P_i$$

(10.29)

second condition can be obtained by the following equation :

$$\sum_{i=1}^{n} P_i - P_D = 0$$

(10.30)

Required Equations for ED solution:

For a known value of λ, the power generated by the i^{th} unit from Eq.(10.29) can be written as:

$$P_i = \frac{\lambda - \beta_i}{2\gamma_i}$$

(10.31)

Eq.(10.31) is valid subject to the satisfaction of equality constraint, which can be written as:

$$\sum_{i=1}^{n} \frac{\lambda - \beta_i}{2\gamma_i} = P_D$$

(10.32)

From the above equation, the required value of λ is:

$$\boxed{\lambda = \frac{p_D + \sum_{i=1}^{n} \frac{\beta_i}{2\gamma_i}}{\sum_{i=1}^{n} \frac{\beta_i}{2\gamma_i}}} \qquad (10.33)$$

The value of λ can be calculated from the above equation and then can be substituted in Eq.(10.31) to compute the values of P_i for i=1,2,….,n. for optimal scheduling of generation.

SOLUTION OF ECONOMIC DISPATCH PROBLEM- NEGLECTING LOSSES AND INCLUDING GENERATOR LIMITS

For satisfactory operation of the thermal unit, power generation by it should not be more than P_{max} or less than P_{min}. This constraint need to be satisfied owing to thermal limits imposed of the unit. Now the problem of economic dispatch is, the minimization of total cost of generation subject to the equality constraint Eq.(10.31) and inequality constraint Eq.(10.34) given below.

$P_i^{\min} \le P_i \le P_i^{\max}$) for i=1,2,…………n

(10.34)

Where P_i^{min} & P_i^{max} are limits specified for unit i.

Following the discussion in the Section 10.4.2, the Kuhn-Tucker necessary conditions that minimize the Lagrange function by including the inequality constraints are:

If generations are with in limits: $\frac{dC_i}{dP_i} = \lambda$ for $P_i^{min} \leq P_i \leq P_i^{max}$

If generations are reached to threshold values: $\frac{dC_i}{dP_i} \leq \lambda$ for $P_i = P_i^{max}$ for $i=1,2,,n.$

If generations are reached less than minimum values: $\frac{dC_i}{dP_i} \geq \lambda$ for $P_i = P_i^{min}$ (10.35)

Numerical Solution:

Estimate the value of λ using Eq.(10.33). Compute corresponding P_i values using Eq.(10.31). Then the equality constraint $\sum P_i = P_D$ is verified subject to the satisfaction of inequality constraints i.e generation limits. If satisfied, λ value is taken to determine optimal generation. Otherwise, λ value must be modified as per the gradient method described below.

Rewrite Eq.(10.32) as :

$f(\lambda) = P_D$

(10.36)

Expand the left hand side of Eq.(10.36) by Taylor's series approximation and neglect higher order terms. Using the r^{th} iteration value of λ,

$$f(\lambda)^{(r)} + \left(\frac{df(t)}{d\lambda}\right)^{(r)} \Delta\lambda^{(r)} = P_D$$

(10.37)

Let $\Delta P^{(r)} = P_D - f(\lambda)^{(r)}$

Eq.(10.37) can be written as:

$$\Delta\lambda^{(r)} = \frac{\Delta P^{(r)}}{\left(\frac{df(\lambda)}{d\lambda}\right)^{(r)}} = \frac{\Delta P^{(r)}}{\sum\left(\frac{dP_i}{d\lambda}\right)^{(r)}}$$

(10.38)

or $\Delta\lambda^{(k)} = \dfrac{\Delta P^{(k)}}{\sum \frac{1}{2\gamma_i}}$

(10.39)

Hence the modified value of λ is :

$$\lambda^{(r+i)} = \lambda^{(r)} + \Delta\lambda^{(r)} \tag{10.40}$$

Where $\Delta P^{(r)} = P_D - f(\lambda)^{(r)} = P_D - \sum_{i=1}^{n} P_i^{(r)}$

As the value of λ is modified, the $\sum P_i$ reaches P_D. If any P_i exceeds maximum or minimum values, then P_i is set to that corresponding limit and maintained constant thereafter. After the P_i is maintained constant, for optimal allocation of load, other units which are not violated limits only must be considered. In other words, all the units other than i must operate at equal incremental costs. This procedure is continued till $\sum P_i$ equals P_D.

ECONOMIC OPERATION OF POWER SYSTEM-2

INTRODUCTION

It is seen in the previous Chapter, the total system load has been optimally divided among the various generating units by equating their incremental generating costs. However, the assumption was power transfer from generating station to load center does not involve transmission line losses. It is unrealistic to neglect transmission losses particularly when long distance transmission of power is involved. The transmission losses vary from 5 to 15 percent of total load. Therefore, it is essential to account for transmission losses while developing an economic load dispatch policy. This Chapter deal with economic scheduling problem by considering transmission line losses.

ECONOMIC SCHEDULING PROBLEM CONSIDERING LOSSES

- STATEMENT OF PROBLEM

The economic dispatch problem is defined as that which minimizes the overall operating cost of all the generators C_T of a power system while meeting the total load P_D plus transmission losses P_L. Mathematically, the problem is defined as follows:

$$C_T = \sum_{i=1}^{n_g} (\gamma_i P_i^2 + \beta_i P_i + \alpha_i) \, Rs/h = \sum_{i=1}^{n_g} C_i (P_i) \qquad (11.1)$$

Subject to the *equality constraint* i.e. energy balance equation given as:

$$\sum_{i=1}^{n_g} P_i = P_D + P_L \qquad (11.2)$$

Where Total active power demand of power system $P_D = \sum_{i=1}^{n_b} P_{Di}$

and the *inequality constraint* i.e. power limits imposed for i^{th} generator.

$P_i^{\min} \leq P_i \leq P_i^{\max}$ (for i=1,2,….,n_g)

Where

α_i , β_i , γ_i are the cost coefficients

n_g = Total number of generating units in power system.

n_b = Total number of buses in power system.

P_{Di} = active power demand at i^{th} bus (for i=1,2,….,n_b).

P_i = real power generation by i^{th} generating unit (for i=1,2,….,n_g).

P_L = Total transmission line power loss.

One of the most important, simple but approximate method of expressing transmission loss as a function of generator power is through B-coefficients. Another more accurate form of transmission loss expression frequently known as the Kron's loss formula

The constrained cost function Eq.(11.1) can be transformed into unconstrained Lagrangian cost function by using Lagrange multiplier that include constraints can be written as:

$$£ = C_T + \lambda(P_D + P_L - \sum_{i=1}^{n_g} P_i) \qquad (11.3)$$

Where λ = Lagrangian multiplier

Kuhn-Tucker condition

Differentiating £ with respect to P_i and equating to zero, gives the condition for optimal operation of the system

$$\frac{d£}{dP_i} = \frac{dC_i}{dP_i} + \lambda(\frac{\partial P_L}{\partial P_i} - 1) = 0$$

$$\frac{dC_i}{dP_i} = (1 - \frac{\partial P_L}{\partial P_i})\lambda \qquad (11.4)$$

$$\frac{dC_i}{dP_i} \times \frac{1}{(1 - \frac{\partial P_L}{\partial P_i})} = \lambda \qquad (11.5)$$

Using Eq.(11.5), the *coordinate equations* for n_g units can be written as:

$$\frac{dC_1}{dP_1}\frac{1}{1 - \frac{\partial P_L}{\partial P_1}} = \frac{dC_2}{dP_2}\frac{1}{1 - \frac{\partial P_L}{\partial P_2}} = \text{.........} = \frac{dC_{n_g}}{dP_{n_g}}\frac{1}{1 - \frac{\partial P_L}{\partial P_{n_g}}} = \lambda \qquad (11.6)$$

The general term in the Eq.(10.6) is written as:

$$L_i \frac{dC_i}{dP_i} = \lambda \qquad \text{(for } i=1,2,..n_g\text{)} \qquad (11.7)$$

Where L_i is known as the penalty factor of i^{th} unit.

$$L_i = \frac{1}{1 - \frac{\partial P_L}{\partial P_i}} \qquad (11.8)$$

Where

$\frac{\partial P_L}{\partial P_i}$ (for i=1,2,..n_g) is defined as the incremental transmission loss (ITL) associated with the i^{th} unit.

λ = Lagrangian multiplier also defined as *incremental cost of received power* units in Rs/MWh.

The Eq.(11.6) indicates, the minimum overall cost of generation can be obtained when the Incremental fuel cost of each unit multiplied by its penalty factor is the same for all the n_g generating units.

The general term in the Eq.(11.6) can be written as:

$$\mathbf{IC_i = \lambda\,[\,1\text{-}ITL_i]} \qquad (11.9)$$

Where

$IC_i = \frac{dC_i}{dP_i}$ = Incremental Cost of Generation of i^{th} unit (for i=1,2,..n_g)

ITL$_i$ = $\frac{\partial P_L}{\partial P_i}$ = incremental transmission loss (ITL) associated with the i^{th} unit. (for i=1,2,..n_g)

The optimum load allocation amongst the units by considering transmission system losses can be obtained by operating all the generators at a particular generation at which Eq.(11.7) i.e. $L_i \frac{dC_i}{dP_i} = \lambda$ for all the units should be the same. The penalty factors L of the units can be determined through incremental transmission loss ITL. To find ITL, we need to develop formula for transmission line loss. The iterative procedure for solving economic scheduling problem will be presented later, the derivation of transmission loss formula is undertaken in the next section.

DERIVATION OF TRANSMISSION LOSS FORMULA

This section presents an easier method for derivation of transmission loss formula. Fig.11.1 represents tv generating units 1 and 2 feeding to number of loads, totaling to I_L through a large transmission network. Out of many branches within the network, consider a branch (any three phase transmission line) designated as b.

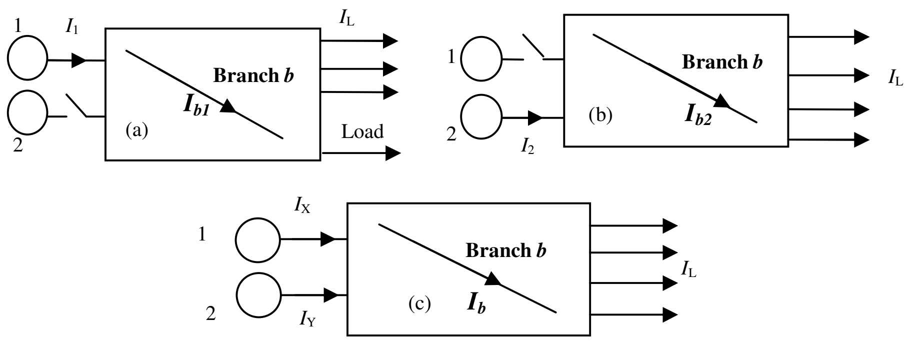


Fig 11.1 Network with two generators, large number of loads and any one branch *b* of network

Let the switch in Fig.11.1(a) is open and the total load current I_L is supplied by generator 1 only say. With generator 1 acting alone, the current in the line b is I_{b1} where as the generator current is I_1 .
Now define current distribution factor a_{b1} as:

$$a_{b1} = \frac{I_{b1}}{I_1} \tag{11.10}$$

Similarly, let the switch in Fig.11.1(b) is open and the total load current I_L is supplied by generator 2 only. With generator 2 acting alone, the current in the line b is I_{b2} where as the generator current is I_2.
Now define current distribution factor a_{b2} as:

$$a_{b2} = \frac{I_{b2}}{I_2} \tag{11.11}$$

When both the generators are connected to supply the load current, the current through the branch *b* i.e. I_b can be obtained by applying the principle of superposition as:

$$I_b = I_{b1}+I_{b2} = a_{b1}\, I_1 + a_{b2}\, I_2 \qquad (11.12)$$

The current distribution factors are taken as the real numbers owing to the following assumptions:

1)The phase angle of all the load currents is the same.

2)The ratio X/R for all the transmission lines in the transmission network is same.

Let $\begin{aligned} I_1 &= |I_1| \angle\theta_1 \\ I_2 &= |I_2| \angle\theta_2 \end{aligned}$

Where θ_1, θ_2 are the phase angles with respect to a common reference.

$I_1 = |I_1| \cos\theta_1 + j\,|I_1| \sin\theta_1$

$I_2 = |I_2| \cos\theta_2 + j\,|I_2| \sin\theta_2$

From Eq.(11.12),

$$I_b = a_{b1} I_1 + a_{b2} I_2 \qquad (11.13)$$

$$I_b = a_{b1} [\,|I_1| \cos\theta_1 + j\,|I_1| \sin\theta_1\,] + a_{b2} [\,|I_2| \cos\theta_2 + j\,|I_2| \sin\theta_2\,]$$

$$= a_{b1} |I_1| \cos\theta_1 + j a_{b1} |I_1| \sin\theta_1 + a_{b2} |I_2| \cos\theta_2 + j a_{b2} |I_2| \sin\theta_2$$

$$= a_{b1} |I_1| \cos\theta_1 + a_{b2} |I_2| \cos\theta_2 + j[\, a_{b1} |I_1| \sin\theta_1 + a_{b2} |I_2| \sin\theta_2\,]$$

$$I_b^2 = a_{b1}^2 |I_1|^2 + a_{b2}^2 |I_2|^2 + 2 a_{b1} a_{b2} |I_1| |I_2| \cos(\theta_1 - \theta_2) \qquad (11.14)$$

Now, let

P_1, P_2 = 3-Phase real power outputs by generators 1 and 2 respectively.

$I_1 = (P_1/\sqrt{3}\times V_1 \text{Cos}\varnothing_1)$; $I_2 = (P_2/\sqrt{3}\times V_2 \text{Cos}\varnothing_2)$

Cos $\varnothing_1$, Cos $\varnothing_2$ = power factors at generator ends.

V_1, V_2= bus voltages at the generator 1 and 2 ends.

R_b is the branch resistance of the branch *b*

k number of branches in the transmission network say.

Now, the total transmission loss in the entire network is given by

$$P_L = 3\sum_{b=1}^{k} I_b^2 R_b \qquad (11.15)$$

Substitute I_b^2 from Eq. (11.14) in Eq.(11.15)

$$P_L = 3\sum_{b=1}^{k} R_b\, [a_{b1}^2 |I_1|^2 + a_{b2}^2 |I_2|^2 + 2 a_{b1} a_{b2} |I_1| |I_2| \cos(\theta_1 - \theta_2)]$$

$$=\sum_{b=1}^{b} 3R_b a_{b1}^2 \mid I_1 \mid^2 + \sum_{b=1}^{b} 3R_b a_{b2}^2 \mid I_2 \mid^2 + \sum_{b=1}^{b} 3R_b 2a_{b1}a_{b2} \mid I_1 \mid\mid I_2 \mid cos(\theta_1 - \theta_2)$$

Substituting I_1 and I_2,

$$=\sum_{b=1}^{k} 3R_b a_{b1}^2 \frac{P_1^2}{3\mid V_1 \mid^2 cos^2 \phi_1} + \sum_{b=1}^{k} 3R_b a_{b2}^2 \frac{P_2^2}{3\mid V_1 \mid^2 cos^2 \phi 2}$$
$$+ \sum_{b=1}^{k} 3R_b 2a_{b1}a_{b2} \frac{P_2 P_1}{3\mid V_1 \mid\mid V_2 \mid cos \ \ \phi_1 cos\phi_2} cos(\theta_1 - \theta_2)$$

$$=\sum_{b=1}^{k} a_{b1}^2 \frac{P_1^2 R_b}{\mid V_1 \mid^2 cos^2 \phi_1} + \sum_{b=1}^{k} a_{b2}^2 \frac{P_2^2 R_b}{\mid V_1 \mid^2 cos^2 \phi_2}$$
$$+ \sum_{b=1}^{k} R_b 2a_{b1}a_{b2} \frac{P_2 P_1}{\mid V_1 \mid\mid V_2 \mid cos \ \ \phi_1 cos\phi_2} cos(\theta_1 - \theta_2)$$

Above loss equation in terms of *B*- coefficients can be written as:

$$P_L = B_{11}P_1^2 + B_{22}P_2^2 + 2B_{12}P_1P_2 \tag{11.16}$$

Where,

$$B_{11} = \sum_{b=1}^{k} a_{b1}^2 \frac{R_b}{\mid V_1 \mid^2 cos^2 \phi_1}$$

$$B_{22} = \sum_{b=1}^{k} a_{b2}^2 \frac{R_b}{\mid V_1 \mid^2 cos^2 \phi_2}$$

$$B_{12} = \sum_{b=1}^{k} \frac{a_{b1} a_{b2} R_b}{\mid V_1 \mid\mid V_2 \mid cos \ \varphi_1 cos\varphi_2} cos(\theta_1 - \theta_2)$$

The terms B_{11}, B_{12} & B_{22} are called *loss coefficients* or *B- coefficients.* The units of *B*-coefficients are in MW^{-1} when the voltages are in KV and branch resistances in ohms.

***B- coefficients* for multi-machine power system**.

Let Eq.(11.16) be extended for a power system with $\boldsymbol{n_g}$ number of generators.

Total transmission losses in the system are:

$$\boldsymbol{P_L = \sum_{m=1}^{n_g} \sum_{n=1}^{n_g} P_m B_{mn} P_n} \tag{11.17}$$

Where B_{mn} are the elements of *B*-Matrix.

Diagonal terms in *B*-Matrix:

$$B_{nn} = \sum_{b=1}^{k} \frac{R_b a_{bn}^2}{\mid V_n \mid^2 cos^2 \phi_n} \tag{11.18}$$

Off-Diagonal terms in *B*-Matrix:

$$B_{mn} = \sum_{b=1}^{k} \frac{a_{bm} a_{bn} R_b}{\mid V_m \mid\mid V_n \mid cos \ \ \phi_m cos\phi_n} \tag{11.19}$$

Expanding Eq.(11.17),

$$P_L = B_{11}P_1^2 + B_{22}P_2^2 + B_{33}P_3^3 + ... + 2P_1P_2B_{12} + 2P_2P_3B_{23} + ... \tag{11.20}$$

In matrix form of Eq.(11.20),

$$P_L = \begin{bmatrix} P_1 & P_2 & \dots & P_n \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} & \dots & B_{1n} \\ B_{21} & B_{22} & \dots & B_{2n} \\ \vdots & \vdots & \dots & \vdots \\ B_{n1} & B_{n2} & \dots & B_{nn} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_n \end{bmatrix} \qquad (11.21)$$

In condensed form of Eq.(11.21),

$$P_L = P\ B\ P^T \qquad (11.22)$$

MODULEII:

HYDRO THERMAL SCHEDULING

Introduction

For optimum utilization of all energy sources in the most economical manner and for the availability of uninterrupted supply, integrated operation of power system is inevitable. At present large power systems are a mix of different modes of generating stations, of which hydro and thermal power stations, integrated operation is predominant. In some systems hydro generation may be more than thermal generation and in some other cases it may be the other way.

In the view of increased demand for electric power, for purposes such as industrial, agricultural, commercial and domestic together with high cost of fuel as well as its limited reserve, considerable attention is being given to hydro thermal coordination problem.

In hydrothermal combined operation, it is necessary to use the total available quantity of water from hydro system to the fullest extent. Fixed charges continue regardless of power generation in case of hydro plants, since no fuel cost is associated. Therefore, minimum overall cost is obtained by optimization of hydro resources. Many of hydro electric plants are multipurpose.

Hydro Thermal Scheduling:

The economic scheduling in the integrated operation is difficult as the combined operation of hydro and thermal plants is subject to a variety of constraints. There are multiple factors that are to be satisfied for their satisfactory operation.

Hydro thermal scheduling is possible with certain assumptions made wherever necessary.

Hydro-Thermal Combined System:

In hydro-thermal integrated operation, hydro generation may be more than thermal generation and in some other cases it may be the other way. However, the exact mode of operation depends upon several factors.

The operating cost of thermal plants is high even though their capital cost is low. In case of hydro electric plants, running costs are very low, but their capital cost is high as construction of dams, canals, penstocks, surge tanks and other elements of development are required in addition to the construction of power house. So, it has become economical and convenient to have both hydro and thermal plants in same grid.

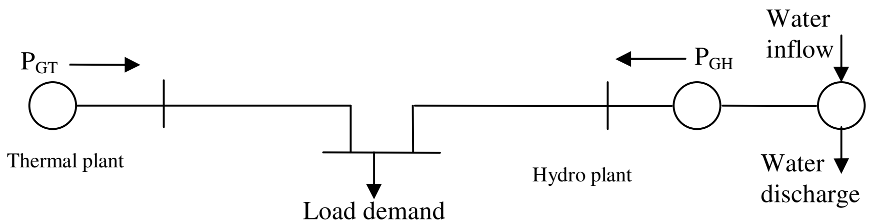


Fig12.1 A basic Hydro-Thermal plant

In case of hydro plants, they can be started easily, they has higher reliability and can be assigned load in very short time. They have greater speed of response. But in case of thermal plants, it requires several hours to bring the boiler, super heater and turbine system ready to take the load. Hence hydro plants can take up fluctuating loads effectively. Because of this reason, thermal plants are preferred as base load plants and hydro plants are operated as peak load plants. The exact operation depends on certain factors like type of development, maximum fore-bay elevation, minimum plant discharge and spillage, storage and pondage, and the amount of water that is available is the most important consideration. Other distinctions among hydro power systems are the number of hydro stations their locations and operating characteristics.

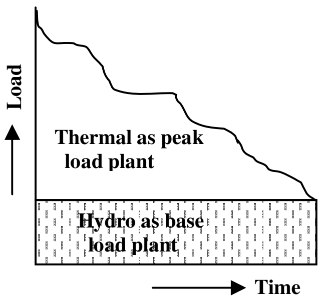


Fig 12.2(a) Hydro plant used as base load plant during normal run-off in an inter connected system

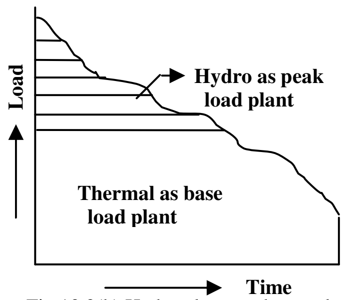


Fig 12.2(b) Hydro plant used as peak load plant during drought period in a inter connected system

Thermal plants can be used at any portion of load duration curve but it is more expensive to use peak load station at low load factors. Besides the insignificant incremental cost in hydro plant, the problem of optimizing the operational cost of a hydro thermal system can be done by minimizing the fuel cost of thermal plants the constraints of availability of water, in a given period for hydro generation.

The cyclic nature of load demand and water inward flow, split the problem into long range scheduling and short range (term/period) scheduling. Here, we consider the short range fixed head problem only.

Hydro Electric Plants:-

Hydro plants are classified based on their type and the location.

Based on their **type** hydro plants are classified into pumped storage plants, run-off river plants and storage plants.

Pumped storage plants: Whenever old thermal stations are available they are generally used to take up peak loads. If such plants are not available then to take up the load it is desirable to develop pumped storage plant for the purpose. A pumped storage plant is associated with upper and lower reservoirs. Such a plant possess the following advantages when used in inter connection.

1) Thermal plants are loaded more economically.
2) The wastage of off-peak energy of thermal plants is reduced.
3) A pumped storage plant stores the energy using off-peak load and supplies when demand rises.

Run of river plants: Run of river plants have very little storage capacity and use water as it becomes available. The water not utilized is spilled. As the variation of run-off during the year does not match the variation of power demand during the year, it becomes necessary to combine run of river plant with steam plant to supply load with maximum reliability.

Storage Plants: These are associated with reservoirs with significant storage capacity. Water is stored during off load requirements and then the stored water is released when demand is high.

On the basis of **location**, hydro plants are classified into three types, a) Cascaded plants, b)Hydro plants on different stream and c) Multi chain hydro plants.

Cascaded plants: These are the plants located on same streams. Down stream plant depends on upstream plant. The down stream plant influences the immediate upstream plant by its effect on tail water elevation and effective head.

Hydro plants on different streams: The hydro plants are located on different streams and are independent of each other.

Multi chain hydro plants: These are located both on different streams and same streams.

Besides the classification of above hydro plants, a number of mathematical models were proposed. Some of them are Glimn-Kirchmayer model, Hildebrand's model, Hamilton-lamonts's model and Arvanitidis-Rosing model. Here we consider Glimn-Kirchmayer model, to solve our problem of optimization.

The rate of water discharge according to Glimn-Kirchmayer model is given as:

$q = k\psi(h)\Phi(P)$

(12.1)

where k is constant of proportionality

h is effective head in meters

P is real power generation in MW

The functions ψ and Φ are defined as quadratic functions,

i.e., $\psi(h)=\alpha h^2+\beta h+\gamma$, where α, β, γ are the coefficients

$\Phi(h)= xP^2+yP+z$, where x, y, z are the coefficients.

Method Of Solution:

The objective of an electric power system is to minimize the total system operating cost, represented by the fuel cost required for the system's thermal generation, over the optimization interval.

The output of each hydro-unit varies with effective head and the rate of water discharged through turbines. For large capacity reservoirs, it is practice to assume the effective head to be constant over optimization interval.

Defining the problem: The problem is to minimize the operating cost of hydro thermal system which can be obtained by minimizing the fuel cost of the thermal plants under the constraints of water availability for hydro generation over a given period of operation.

Variables:

$F_i(P_i(t))$ –Fuel cost of the i^{th} thermal unit in Rs/h

$q_j(t)$ – Discharge of water of j^{th} hydro unit in m^3/h

T– Optimization interval

J– Minimization function

N_T – No of thermal units

N_H – No of hydro units

a_i, b_i, c_i – cost coefficients for i^{th} unit

x_i, y_i, z_i –discharge coefficients for i[th] unit

B_{ij} – loss cofficients

P_{DK} – Demand during k[th] interval

V_j – Prescribed volume of water for j[th] unit.

$P_L(t)$ – Transmission loss during the sub interval

w_j – Water conversion factors

t_k – Duration of k[th] interval

Consider, an electric power system network having N_H hydro power plants, N_T thermal generating plants and therefore N_H+N_T is the total number of generating plants.

Stage1:- Considering ***thermal model***, over the optimization interval 0 to *T*, the function to be minimized is the total system operating cost, represented by fuel cost of thermal generation.

\

$$J = \int_0^T \sum_{i=1}^{N_T} F_i(P_i(t))dt$$

(12.2)

Where $F_i\ (P_i(t))$ is fuel cost of i[th] thermal unit and fuel cost is a function of active power generation of that particular unit, and is approximated by

$$F_i\ (P_i(t)) = a_i P_i^2(t) + b_i P_i(t) + c_i \quad (i=1 \text{ to } N_T)$$

(12.3)

Where a_i, b_i and c_i are cost coefficients of i[th] generating unit.

Stage2:- Considering ***hydro model***, the operation of hydro plants depends on the availability of water. The variation of water discharge with time q(t) is expressed as function of output power P(t) and net head h. According to glimn-kirchmayer model in the earlier section, discharge is

$$q_j(t) = k\ \psi(h_j)\ \Phi(P_{j+N_T}\ (t)) \quad \text{m}^3/\text{h} \qquad (j=1 \text{ to } N_H)$$

(12.4)

Where ψ *and* Φ are two independent functions.

In case of constant head $\psi(h_j)$ becomes constant, so the above equation is rewritten as

$$q_j(t) = k'\ \ \Phi(P_{j+N_T}\ (t)) \text{ and where } k' = k\ \psi(h_j) \quad (j=1 \text{ to } N_H)$$

(12.5)

Each hydro plant is constrained by the amount of water available for the optimization interval,

$$\int_0^T q_j(t)dt = V_j \qquad (j=1 \text{ to } N_H)$$

(12.6)

Where V_j is predefined volume of water in m^3 and

$$q_j(t) = x_j P_{j+N_T}^2(t) + y_j P_{j+N_T} + z_j \quad \text{m}^3/\text{h} \quad ;$$

Where $(j=1 \text{ to } N_H)$; x_j, y_j, z_j are discharge coefficients of j^{th} hydro plant

Stage3:- Considering equality and inequality constraints, load demand equality constraint is

$$\sum_{i=1}^{N_T+N_H} P_i(t) = P_D(t) + P_L(t)$$

(12.7)

Where $P_D(t)$ is load demand and $P_L(t)$ is the transmission loss during the sub-interval and the limits of power generation at the i^{th} bus are imposed as

$$P_i^{min} \le P_i(t) \le P_i^{max} \quad ; \quad (i=1 \text{ to } N_H+N_T)$$

(12.8)

where P_i^{min} is lower limit & P_i^{max} is the upper limit of the i^{th} generator output.

Stage4:- Approximated loss formula in terms of loss coefficients is

$$P_L(t) = \sum_{i=1}^{N_T+N_H} P_i(t) B_{ij} P_j(t) + \sum_{i=1}^{N_T+N_H} B_{i0} P_i(t) + B_{00} \quad \text{MW}$$

(12.9)

Where B_{00}, B_{i0}, B_{ij} are B-coefficients or loss coefficients

Let us introduce an unknown multiplier function $\lambda(t)$, known as lagrange multiplier, so the required equation is transformed in terms of lagrange multiplier into

$$(P_i(t), \lambda(t), w_j) = \int_0^T [\sum_{i=1}^{N_T} F_i(P_i(t)) + \sum_{j=1}^{N_H} w_j q_j(t) + \lambda(t)[P_D(t) + P_L(t) - \sum_{i=1}^{N_T+N_H} P_i(t)]]dt - \sum_{j=1}^{N_H} w_j V_j$$

(12.10)

where $\lambda(t)$ is incremental cost of power delivered in system and w_j are water conversion factors.

The optimum condition can be described by taking partial derivatives of augmented objective function with respect to the $P_i(t)$, $\lambda(t)$ and w_j. where $(i=1 \text{ to } N+M)$ & $(j=1 \text{ to } M)$

$$\frac{\partial F_i(P_i(t))}{\partial P_i(t)} + \lambda(t)\left[\frac{\partial P_L(t)}{\partial P_i(t)} - 1\right] = 0 \qquad (i=1 \text{ to } N_T)$$

(12.11a)

$$w_j \frac{\partial q_j(t)}{\partial P_{j+N_T}(t)} + \lambda(t)\left[\frac{\partial P_L(t)}{\partial P_{j+N_T}(t)} - 1\right] = 0 \qquad (j=1 \text{ to } N_H)$$

(12.11b)

and $\int_0^T q_j(t)dt = V_j \qquad (j=1 \text{ to } N_H)$

(12.11c)

$$\sum_{i=1}^{N_T+N_H} P_i(t) = P_D(t) + P_L(t)$$

(12.11d)

Stage5:- The problem of optimization can be redefined in discrete form as

Minimize $J = \sum_{k=1}^{T}\sum_{i=1}^{N_T} t_k F_i(P_{ik})$

(12.12a)

Subject to $\sum_{i=1}^{N_T+N_H} P_{ik} = P_{Dk} + P_{Lk}$; *(k=1,2,….T)*

(12.12b)

$\sum_{k=1}^{T} t_k q_{jk} = V_j$; $(j=1,2,\ldots.N_H)$

(12.12c)

$P_i^{\min} \le P_{ik} \le P_i^{\max}$ $(i=1 \text{ to } N_T \ \& \ k=1 \text{ to } T)$

(12.12d)

In the time interval k, the cost function $F_i(P_{ik})$ of thermal units in its discrete form is defined by

$F_i\,(P_{ik}) = a_i P_{ik}^2 + b_i P_{ik} + c_i$ Rs/h

(12.13)

with a_i, b_i and c_i as cost coefficients P_{ik} is output of thermal and hydro units during the k^{th} interval.

Transmission losses during k^{th} interval P_{LK} is given by

$P_{LK} = \sum_{i=1}^{N_T+N_H}\sum_{j=1}^{N_T+N_H} P_{ik}B_{ij}P_{jk} + \sum_{i=1}^{N_T+N_H} B_{i0}P_{ik} + B_{00}$ MW

(12.14)

B_{ij}, B_{i0}, B_{00} are loss coefficients

P_{DK} is demand during k^{th} interval.

$q_{jk} = x_j P_{j+N_T,k}^2 + y_j P_{j+N_T,k} + z_j$ m^3/h

(12.15)

Where q_{jk} is rate of discharge from j^{th} hydro unit in interval k. x_j, y_j, z_j are discharge coefficients of j^{th} units as described earlier.

The above objective by the constraints is given below,

$$L(P_{ik}, \lambda_k, w_j) = \sum_{k=1}^{T}\left[\sum_{i=1}^{N_T} t_k F_i(P_{ik}) + \sum_{j=1}^{N_H} w_j t_k q_{jk} + \lambda_k\left[P_{DK} + P_{LK} - \sum_{i=1}^{N_T+N_H} P_{ik}\right]\right] - \sum_{j=1}^{N_H} w_j V_j$$

(12.16)

Where V_j is prespecified volume of water available w_j is water conversion factor.

By taking partial derivatives of augmented objective function with respect to P_{ik}, λ_k & w_j we can obtain optimal condition.

$$t_k \frac{\partial F_i}{\partial P_{ik}} + \lambda_k \left[\frac{\partial P_{LK}}{\partial P_{iK}} - 1\right] = 0$$; (i=1,2,….N_H) & (k=1,2,....T)

(12.17a)

$$w_j t_k \frac{\partial q_{ik}}{\partial P_{mk}} + \lambda_k \left[\frac{\partial P_{LK}}{\partial P_{mK}} - 1\right] = 0$$; *(j=1 toN_H, m=N_T+j, K=1 to T)*

(12.17b)

$$\sum_{k=1}^{T} t_k q_{jk} = V_j$$; *(j=1 to N_H)*

(12.17c)

$$\sum_{i=1}^{N_T+N_H} P_{ik} = P_{DK} + P_{LK}$$; *(k=1 to T)*

(12.17d)

The above equations are non linear, so by introducing small changes in P_{ik} & λ_k. we obtain the equations 12.18a, 12.18b, 12.18c, 12.18d as

$$\left(t_k \frac{\partial F_i^2}{\partial P_{ik}^2} + \lambda_k \frac{\partial^2 P_{LK}}{\partial P_{ik}^2}\right)\Delta P_{ik} + \lambda_k \sum_{\substack{j=1 \\ i \neq 1}}^{N_T+N_H} \frac{\partial^2 P_{LK}}{\partial P_{ik}\partial P_{jk}} \Delta P_{jk} + \left(\frac{\partial P_{LK}}{\partial P_{iK}} - 1\right)\Delta\lambda_k = -\left[t_k \frac{\partial F_i}{\partial P_{ik}} + \lambda_k \left(\frac{\partial P_{LK}}{\partial P_{iK}} - 1\right)\right]$$ *; (i=1 to N_T)*

(12.18a)

$$\left(w_j t_k \frac{\partial^2 q_{ik}}{\partial P_{mk}^2} + \lambda_k \frac{\partial^2 P_{LK}}{\partial P_{iK}^2}\right)\Delta P_{mk} + \lambda_k \sum_{\substack{n=1 \\ n \neq m}}^{N_T+N_H} \frac{\partial^2 P_{LK}}{\partial P_{mk}\partial P_{nk}} \Delta P_{nk} + \left(\frac{\partial P_{LK}}{\partial P_{mk}} - 1\right)\Delta\lambda_k = -\left[w_j t_k \frac{\partial q_{jk}}{\partial P_{mk}} + \lambda_k \left(\frac{\partial P_{LK}}{\partial P_{mk}} - 1\right)\right]$$; *(j=1 to N_H, m=j+N_T)*

(12.18b)

$$\sum_{j=1}^{N_T+N_H} \left(\frac{\partial P_{LK}}{\partial P_{jK}} - 1\right)\Delta p_{jk} = -\left(P_{DK} + P_{LK} - \sum_{i=1}^{N_T+N_H} P_{ik}\right)$$

(12.18c)

By substituting values of partial derivatives with initial values of control variables, in eq (12.18)

$$(2t_k a_i + \lambda_k^0 B_{ii})\Delta P_{ik} + \sum_{\substack{j=1 \\ j \neq i}}^{N_t+N_H} 2B_{ij}\Delta P_{jk} + \left(\sum_{j=1}^{N_T+N_H}\left(2B_{ij}P_{jk}^0 + B_{i0} - 1\right)\right)\Delta\lambda_k = -\left[t_k(2a_i P_{ik}^0 + b_i) + \lambda_k^0\left(\sum_{j=1}^{N_T+N_H} 2B_{ij}P_{jk}^0 + B_{i0} - 1\right)\right]$$

(12.19a)

$$(2w_j^0 t_k x_j + \lambda_k^0 B_{mm})\Delta P_{mk} + \sum_{\substack{l=1 \\ l \neq m}}^{N_T+N_H} 2B_{ml}\Delta P_{jl} + \left(\sum_{l=1}^{N_T+N_H} 2B_{ml}P_{lk}^0 + B_{m0} - 1\right)\Delta k = -\left[V_j^0 t_k\left(2x_j P_{mk}^0 + y_j\right) + \lambda_k^0\left(\sum_{l=1}^{N_T+N_H} 2B_{ml}P_{lk}^0 + B_{m0} - 1\right)\right]$$

(j=1 to N_H, m=j+N_T)

(12.19b)

and $$\sum_{j=1}^{N_T+N_H}\left(\sum_{l=1}^{N_T+N_H} 2B_{jl}P_{LK}^0 + B_{j0} - 1\right)\Delta P_{jk} = -\left(P_{DK} + P_{LK}^0 - \sum_{i=1}^{N_T+N_H} P_{ik}^0\right)$$

(12.19c)

The above equations can be written in matrix form as

$$\begin{bmatrix} \Delta_{pp}^k & \Delta_{pk}^k \\ (\Delta_{p\lambda}^k)^T & 0 \end{bmatrix}\begin{bmatrix} \Delta P_k \\ \Delta\lambda_k \end{bmatrix} = \begin{bmatrix} -\Delta_p^k \\ -\Delta_\lambda^k \end{bmatrix}$$

(12.20)

Where s- size of Hessian matrix, $s=N_T+N_H+1$.

Initial Guess:

Let the power demand is equally distributed among thermal & hydro units during each interval,

$$P_{ik}^0 = \frac{P_{DK}}{N_T+N_H} \qquad (i=1 \text{ to } N_T+N_H \;\&\; k=1 \text{ to } T)$$

(12.21)

Further let us assume that there are no transmission losses.

$$\lambda_k^0 = 2a_i P_{ik}^0 + b_i \qquad (k=1,2,\ldots.T)$$

(12.22)

$$w_j^0(2x_j P_{mk}^0 + y_j) = \lambda_k^0 \qquad (j=1 \text{ to } N_H)$$

So water conversion factor is obtained as

$$w_j^0 = \frac{\lambda_k^0}{2x_j P_{mk}^0 + y_j} \qquad (j=1 \text{ to } N_H,\ m=j+N_T)$$

(12.23) **Algorithm:**

Step 1: Read the number of thermal units *N*, hydro units *M*, number of sub intervals *T*, cost coefficients a_i, b_i, c_i, $(i=1 \text{ to } N_T)$ loss coefficients $B_{ij}(i=1 \text{ to } N_T+N_H,\ j=1 \text{ to } N_T+N_H)$ discharge coefficients x_i, y_i, z_i, $(i=1 \text{ to } N_T)$, demand P_{DK} $(k=1 \text{ to } T)$ and pre specified available water V_j $(j=1 \text{ to } N_H)$, ε-error

Step 2: Calculate initial guess value P_{ik}^0 $(i=1 \text{ to } N_T+N_H,\ k=1 \text{ to } T)$, λ_k^0 & w_j^0 $(j=1 \text{ to } N_H)$.

Step 3: consider w_j^0 $(j=1 \text{ to } N_H)$ in step 2 and start the iteration count $c=1$.

Step 4: start hourly count $k=1$.

Step 5: consider $P_{ik}^0 (i=1 \text{ to } N_T+N_H)$ and λ_k^0

Step 6: calculate $\Delta P_{ik} (i=1,2,\ldots.\ N_T+N_H)$ and $\Delta\lambda_k$ and solve equations using one of the iterative methods.

$$\begin{bmatrix} \Delta_{pp}^{k} & \Delta_{pk}^{k} \\ (\Delta_{p\lambda}^{k})^{T} & 0 \end{bmatrix} \begin{bmatrix} \Delta P_{k} \\ \Delta \lambda_{k} \end{bmatrix} = \begin{bmatrix} -\Delta_{p}^{k} \\ -\Delta_{\lambda}^{k} \end{bmatrix}$$

Step 7: check for convergence if, $\left| \sum_{i=1}^{N_T+N_H} \Delta P_{ik} + \Delta \lambda_k \right| \le \varepsilon$ then goto step 9

Step 8: calculate new values of P_{ik}*(i=1 to N+M)* and λ_k. i.e, P_{ik}^{new} and λ_k^{new}.

$$P_{ik}^{new} = P_{ik}^{0} + \Delta P_{ik} \quad \text{and} \quad \lambda_k^{new} = \lambda_k^{0} + \Delta \lambda_k$$

(12.24)

Step 9: set limits correspondingly at

$$P_{ik}^{new} = \begin{Bmatrix} P_i^{\max}; if P_{ik}^{new} \ge P_i^{\max} \\ P_i^{\min}; if P_{ik}^{new} \le P_i^{\min} \\ P_{ik}^{new}; otherwise \end{Bmatrix}$$

(12.25)

If the limits have been set either lower or upper, then set that generator aside, i.e. this generator is not allowed to participate, by deleting i^{th} row and i^{th} column.

Step 10: set $P_{ik}^{0} = P_{ik}^{new}$ *(i=1 to N_T+N_H)* $\lambda_k^{0} = \lambda_k^{new}$ goto step 6 and repeat.

Step 11: If $k \ge T$ then goto step 12 else $k=k+t_k$ and goto step 5

Step 12: calculate V_j *(j=1 to N_H)*

Step 13: If $|V_j - V_j^s| \le \varepsilon$ or if $(c \ge R)$ then goto step 14

else $w_j^{new} = w_j^{0} + (V_j - V_j^s)/V_j^s$,

$w_j^{0} = w_j^{new}$ *(i=1 to N_H)*

$c=c+1$ goto step 4 and repeat.

Step 14: calculate the optimal cost and loss.

Step 15: stop

Module:III

LOAD FREQUENCY CONTROL I

1 INTRODUCTION

In general both active and reactive power demands are continually changes with the rising or falling of load. In order to meet the active power demand, input to the generators i.e. steam for turbo-generators and water for the hydro-generators must be regulated. Otherwise the machine speed will vary with consequent change in frequency, which may be highly undesirable. Similarly, the excitation of generators must be continuously regulated to meet the reactive power demand. Otherwise the voltage at various system buses may go beyond the prescribed limits. In large interconnected systems, automatic generation and voltage regulation equipment is installed for each generator because manual regulation is not feasible. Changes in active power is dependent on internal machine angle δ and is independent of bus voltage; while the bus voltage is dependent on machine excitation and is independent of machine angle δ . Change in angle δ is caused by momentary change in generator speed. Speed of the generator can be regulated by using *Governor Controller* which basically control inlet position to allow more or to reduce steam or water jet hitting the blades of turbine. Similarly, the rise or fall in the terminal voltage can be controlled by *Excitation Controller* which reduces excitation to alternator (when terminal voltage is high) or increase excitation (when terminal voltage is low). The terminal voltage variations are mainly due to variations in reactive power demand.

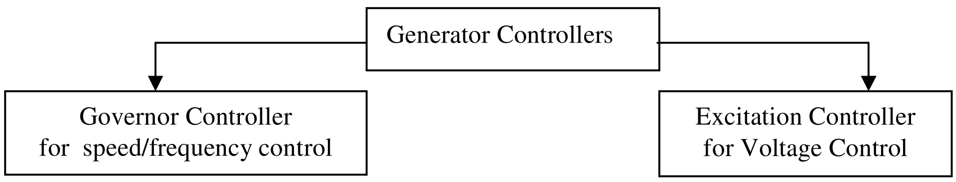


Fig 13.1 Generator Controllers

Excitation voltage control is fast acting because its major time constant encountered due to that of generator field while the governor control is slow acting with major time constant is contributed by the turbine and generator moment of inertia. Due to high difference in the time constants, the two controllers can be analyzed by decoupling them.

This chapter provides mathematical models of turbine, speed governing system, excitation system, generator, loads etc. These models are required for steady state analysis of generator in coming chapters.

13.2 MODELLING OF TURBINE SPEED-GOVERNOR CONTROLLER

Fig. 13.2 is the schematic representation of Turbine Speed Governing system. It has mainly four major components.

Speed governor: Speed governor senses the change in speed (or frequency) hence it can be regarded as heart of the system. The standard model of speed governor operates by fly-ball mechanism. Fly-balls moves outward when speed increases and the point Q on the linkage mechanism moves downwards. the reverse happens when the speed decreases. The movement of point Q is proportional to change in shaft speed.

Linage mechanism: PQR is a rigid link pivoted at Q and RST is another rigid link pivoted at S. This link mechanism provides a movement to the control valve in proportion to change in speed. It also provides a feedback from the steam valve movement.

Hydraulic amplifier: It comprises a pilot valve and main piston arrangement. It converts low power level pilot valve movement into high power level piston valve movement. This is necessary in order to open or close the steam valve against high pressure steam.

Speed changer: It provides a steady state power output setting for the turbine. Its downwards movement opens the upper pilot valve so that more steam is admitted to the turbine under steady conditions the reverse happens for upward movement of speed changer. By adjusting the linkage position of point P the scheduled speed/frequency can be obtained at the given loading condition.

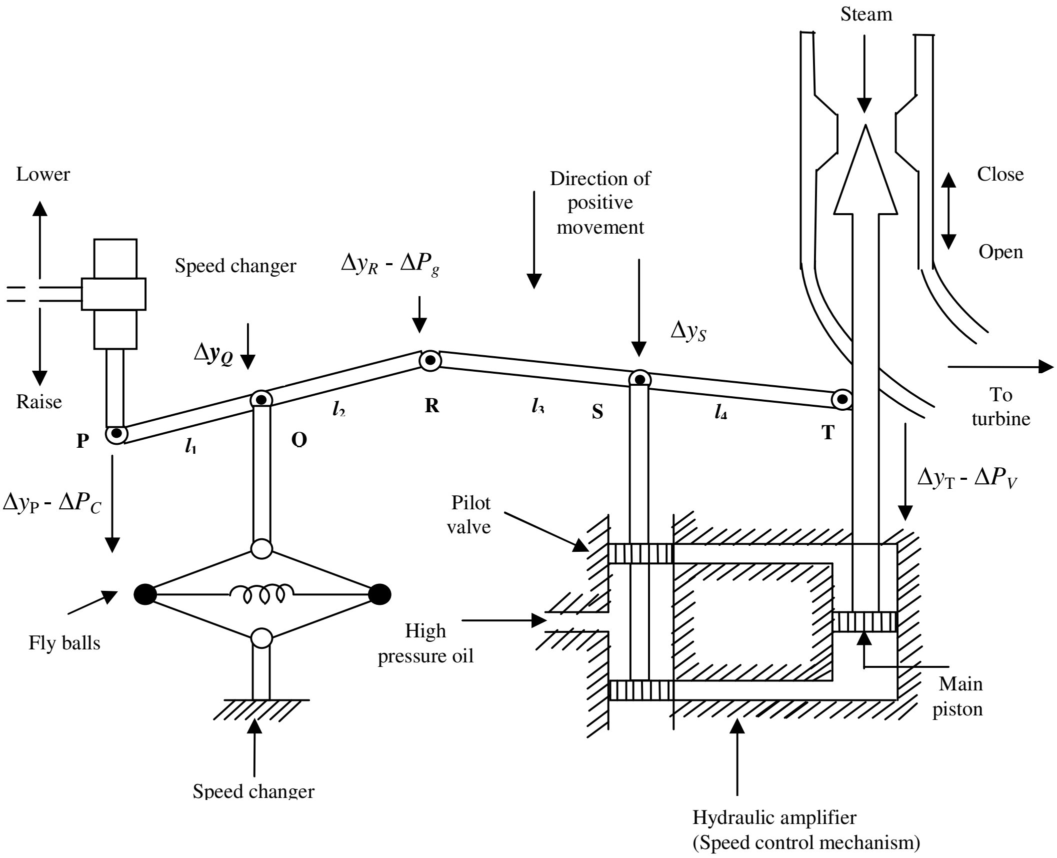


Fig 13.2 Turbine speed governing system

Mathematical Model of Speed Governing System

Assume that the system is initially operating under steady conditions the linkage mechanism stationary and pilot valve closed. Steam valve opened by a definite magnitude, turbine running at constant speed with turbine power output balancing the generator load

Let the operating conditions be characterized by

f^0 system frequency; p_G^0 generator output; y_T^0 steam valve setting

Let the point P on the linkage mechanism be moved downwards by a small amount Δy_P. It is a command which causes the turbine power output to change and can therefore be written as:

$\Delta y_P = k_c \ \Delta P_c$ where ΔP_c is the commanded increase in power

the command signal ΔP_c (i.e. Δy_T) sets into motion a sequence of events .the pilot valve moves upwards, high pressure oil flows on to the top of the main piston moving it downwards; the steam valve opening consequently increases, turbine generator speed increases, i.e. the frequency goes up let us model these events mathematically.

There are two factors contribute to the movement of R:

i) Δy_P Contributes ($(\frac{-l_2}{l_1})\Delta y_p$ (or) $-k_1 \Delta y_P$ (i.e upwards) of $-k_1 k_c \Delta P_c$

ii) Increase in frequency Δf causes the fly-balls to move outwards so that Q moves downwards by a proportional amount $k_2^{'} \ \Delta f$,the consequent movement of R with P remaining fixed at Δy_P is

$$+\frac{l_1 + l_2}{l_1} k_2^{'} \Delta f = +k_2 \Delta f \quad \text{(i.e. downwards)}$$

The net movement of R is therefore

$$\Delta y_R = -k_1 k_c \Delta p_c + k_2 \Delta f$$

(13.1)

The movement of S, Δy_S is the amount by which the pilot valve opens. it is contributed by Δy_R and Δy_T can be written as

$$\Delta y_S = (\frac{l_4}{l_3 + l_4})\Delta y_R + (\frac{l_3}{l_3 + l_4})\Delta y_T = k_3 \Delta y_R + k_4 \Delta y_T$$

(13.2)

the movement Δy_S depending upon its sign opens on of the ports of the pilot valve admitting high pressure oil into the cylinder there by moving the main piston and opening the steam valve by Δy_T .the following justifiable assumptions are made

i) internal reaction forces of main piston and steam valve are negligible compared to the forces exerted on the piston by high pressure oil.

ii)because of (i) above, the rate of oil admitted to the cylinder is proportional to port opening Δy_S the volume of oil admitted to the cylinder is thus proportional to the time integral of Δy_S. the movement Δy_T is obtained by dividing the oil volume by the area of the cross-section of the piston. Thus

$$\Delta y_T = k_S \int_0^t (-\Delta y_S) dt$$

(13.3)

From the above diagram, we can conclude that a positive movement Δy_S ,cause negative (upward) movement Δy_T accounting for the negative sign used in the above example.

Tacking the Laplace transform of Eqns. (1),(2),(3)

We get $\Delta Y_R(s) = -k_1 k_c \Delta P_R(s) + k_2 \Delta F(s)$

(13.4)

$$\Delta Y_S(s) = k_3 \Delta Y_c(s) + k_4 \Delta Y_T(s)$$

(13.5)

$$\Delta Y_T(s) = -k_5 \frac{1}{s} \Delta Y_S(s)$$

(13.6)

Substituting (4) in (5) then in (6) we get by simplification

$$\Delta Y_T(s) = \frac{k_1 k_3 k_c \Delta P_c(s) - k_2 k_3 \Delta F(s)}{(k_4 + \frac{s}{k_5})}$$

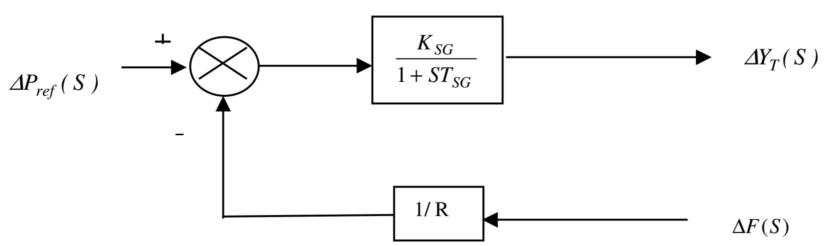
$$= [\Delta P_c(s) - \frac{1}{R}\Delta F(s)] * (\frac{k_{sg}}{1 + sT_{sg}})$$

(13.7)

Where $R = \frac{k_1 k_c}{k_2}$ = governor speed regulation, $K_{sg} = \frac{k_1 k_3 k_c}{k_4}$ = gain of speed governor

$T_{sg} = \frac{1}{k_4 k_5}$ =speed governor time constant

Block diagram representation of Eq.(7)

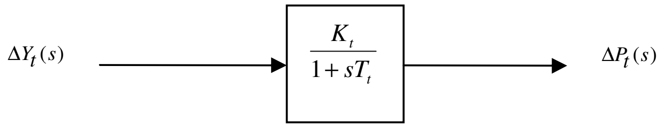


13.4 MODELLING OF STEAM TURBINE

The dynamic response of a steam turbine is largely influenced by two factors

i)entrained steam between the inlet steam valve and first of the turbine,
ii)the storage action in the re heater which causes the outlet of the low pressure stage to lag behind that of the high pressure stage.

Dynamic response of a steam turbine can be related in terms of changes in steam valve opening Δy_T .the following figure shows the transfer function model of steam turbine



The typical values of the turbine time constant T_t lies in the range of 0.2 to 2.5s

13.5 GENERATOR LOAD MODEL

The increment in power input to the generator load system is ΔP_G - ΔP_D where $\Delta P_G = \Delta P_t$, incremental power output and ΔP_D is the load increment. Assume that generator incremental losses are neglected. This increment in power input to the system is accounted for in two ways:

i) Rate of increase of stored kinetic energy in the generator rotor at scheduled frequency f^0

The stored KE is $W^o_{ke} = H*P_r kw$(kilojoules)

Where P_r is the kw rating of the turbo-generator and H is defined as its inertia constant.

The kinetic energy being proportional to square of speed(frequency),the kinetic energy at a frequency of ($f^0 + \Delta f^0$) is given by

$$W_{ke} = W_{ke}^{\ 0}\left(\frac{f^0+\Delta f}{f^0}\right)^2 \cong W_{ke}^0\left(1+2\frac{\Delta f^0}{f^0}\right)$$

and rate of change of KE is therefore

$$\frac{d(W_{ke})}{dt} = 2\frac{HP_r}{f^o}\frac{d(\Delta f)}{dt}$$

ii)as the frequency changes, the motor load changes being sensitive to speed the rate of change of load with respect to frequency i.e ($\frac{\partial P_D}{\partial f}$) can be regulated as nearly constant for small changes in frequency Δf and can be expressed as ($\frac{\partial P_D}{\partial f}$) $\Delta f = B\,\Delta f$ where the constant B can be determined empirically B is positive for a predominantly motor load. Writing the power balance equation, we have

$$\Delta P_G - \Delta P_D = \frac{2HP_r}{f^0}\frac{d(\Delta f)}{dt} + B\Delta f$$

dividing through out by P_r and rearranging, we get

$$\Delta P_G - \Delta P_D = \frac{2H}{f^0}\frac{d(\Delta f)}{dt} + B(p.u)\Delta f$$

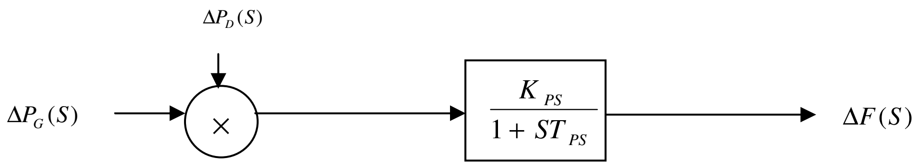
taking the laplace transform, we can write Δf (s)as

$$\Delta F(S) = \frac{\Delta P_G(S) - \Delta P_D(S)}{B + \frac{2H}{f^0}S} = (\Delta P_G(S) - \Delta P_D(S))(\frac{K_{ps}}{1 + ST_{ps}})$$

where $T_{ps} = \frac{2H}{Bf^0}$ = power system time constant

$K_{ps} = \frac{1}{B}$ = power system gain

above equation can be represented in block



13.6 Representation of loads

The loads generally consists of industrial and domestic components the magnitude of the load changes continuously so that the load forecasting problem is truly a statistical one. An industrial load consists mainly of large three phase induction motors with sufficient load constancy and predictable duty cycle where as the domestic load mainly consists of lighting, heating and many single-phase devices used in a random way by householders. the design and operation of power systems both economically and electrically are greatly influenced by the nature and magnitude of loads in general there are three ways of load representation they are

(i) Constant power representation: this is used in load flow studies both specified MW and MVAR are taken to be constant.

(ii) Constant current representation: the load current equation can be written as

$I = \frac{P - jQ}{V^*} = |I|\angle(\delta - \theta)$ Where $V = |V|\angle\delta$ $\theta = \tan^{-1}(\frac{Q}{P})$ is the power factor angle .It is known as constant current representation because the magnitude of current is neglected as constant current representation because the magnitude of current is regarded as constant.

iii) Constant impedance representation: this representation is generally used in stability studies. the load specified in MW and MVAR at nominal voltage is used to compute the load impedance. Thus

$$Z = \frac{V}{I} = \frac{VV^*}{P - jQ} = \frac{|V|^2}{P - jQ} = \frac{1}{Y}$$

This then is regarded as constant throughout the study.

13.7 TURBINE MODEL

For a simple non-reheat type turbine the model is given by a single time constant but reheat turbines have more than one time constant. If a reheat unit has two stage stem turbines thus the dynamic response will be influenced by

1) The entrained steam between the steam inlet valve and first stage of turbine and
2) The storage action in the reheated which causes the outlet of the L.P stage to leg behind that of the H.P stage

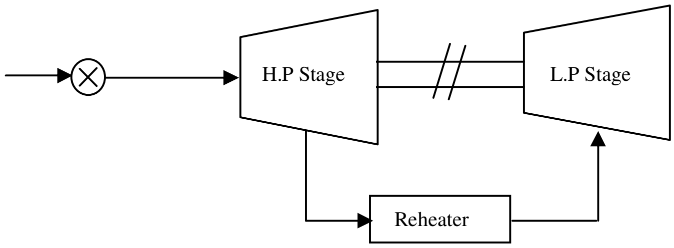
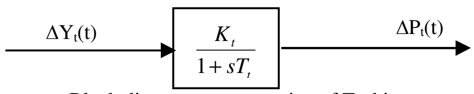


Fig 13.2 two stage reheater unit

The turbine transfer function in characterized by two time constants. For case of analysis it can be assumed that the turbine in modeled by a single equivalent time constant T_t which valve lies between 0.2 to .25 sec.



Block diagram representation of Turbine

SINGLE AREA LOAD FREQUENCY CONTROL

INTRODUCTION

In power system the load demand is continuously changing. In accordance with it power input also varies.

If the input – output balance is not maintained a change in frequency occur. So the frequency of control can be achieved firstly through speed governor mechanism. The basic characteristics of governor are relation between speed and load. If load on turbine increases, the speed of the governor decreases. The frequency normally vary by 5% between light load and full load conditions. Consider two machines both are running under parallel n as shown 14.1. load sharing between two machines as follows.

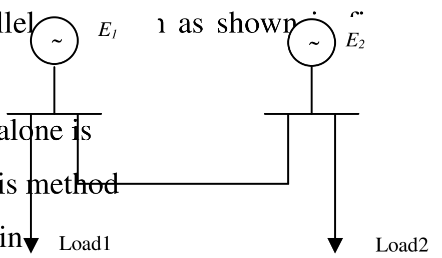


Fig 14.1

If the change in load either at E_1 or E_2 and if the generation of E_2 alone is regulated to adjust this change so as to have constant frequency, this method of regulation is known as Parallel frequency regulation. If change in in a particular area is taken care by the generator in that area the tie-line loading remains constant. This is known as flat tie-line loading control.

In large Inter-connected power system manual regulation is not feasible

Important control loops in power system are

1. Frequency Control

2. Automatic Voltage Control

In this chapter the frequency control will be discussed. Necessity for keeping frequency constant:

1. In large interconnected power system network the real and reactive power are continuously changing with a falling and rising loads but never maintain constant,
2. Frequency and voltage of system are maintained at their normal values by monitoring the load variations and using suitable control action to match the real and reactive power generations with the load demand and the losses in the system at the time.
3. The system frequency is closely related to the real power balance in the power system network the system frequency is mainly controlled by the real power balance in the system.
4. Load increase on generating unit, more amount of real is to supplied which is immediately received from the kinetic energy power in rotating part thereby reducing the kinetic energy of angular velocity or speed of the machine.
5. Voltage is controlled by reactive power control in system.

Load frequency and excitation voltage regulators of turbo generators as shown in fig 14.2. these controllers are set for particular operating condition they take of small changes in load with out exceeding   the prescribed values change in load becomes large. The controllers must be reset either manually or automatically.

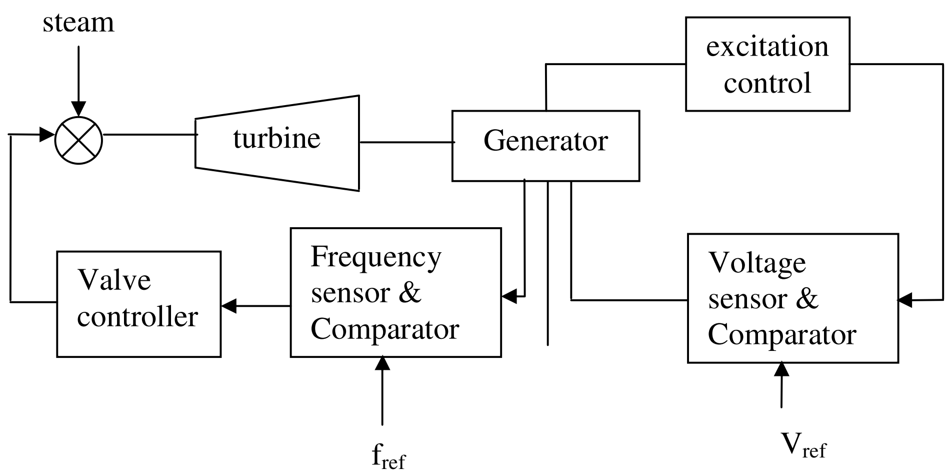


Fig 14.2

Considering problem of controlled the power output of generator of a closely connected area so as to maintain the constant frequency.

Control area concept

In the olden days electric power systems were usually operated as individual units. Due to demand for large blocks of power and increased reliability, interconnection of neighboring plants. it's advantages economically because four machines are required as reserve for operation at peak loads (reserve capacity) and fewer machines running without load are required to take core of sudden unexpected jumps in load(spinning reverse). There fore, all generating plants are interconnected to form a state grid , regional grid and national grid. Load dispatch centers are required for the control of power flow in grid.

It is feasible to divide a very large power system ,say a national grid into sub areas in which all the generators are assumed to tightly coupled , i.e. they swing in unison with change in load or due to speed changer setting . such an area, where all the generators running coherently, is termed as

All generators in this area constitute a coherent group. So that all generated speed up or shutdown simultaneously such an area is known as control area.

Consider a single turbo generator system supply an isolated load. Speed governing system controls the real power flow in the power system.:

Isolated block diagram representation single area frequency control

(1) Speed Governing System:

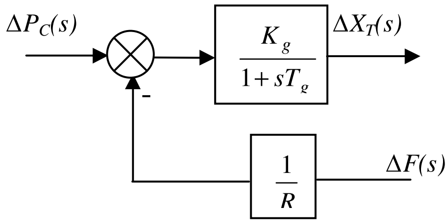
K_G = Governor gain constant

T_G = Governor time constant

ΔP_C (s) =speed changer setting

$\Delta X_T(s)$ =change in position at point T

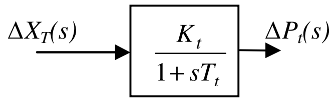
In linkage mechanism



(a)

(2) Turbine model representation:

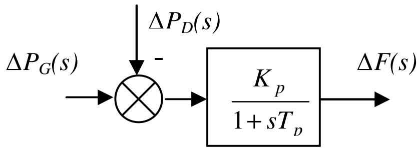
K_t = Turbine gain constant



(b)

T_t = Turbine time constant

(3) Generator load model representation:



(c)
Fig 14.3

Models for turbine, speed governor power system are obtained. In practice rarely a single generator feeds a large area. Several generators are connected in parallel, located also at different places will supply the power needs of a geographical area. Quite normally, all these generators may have the same response characteristics for the changes in load demand. In such case, it is called as control area.

By combining all fig 14.3 (a,b,c) the above block diagrams representation for single area control.

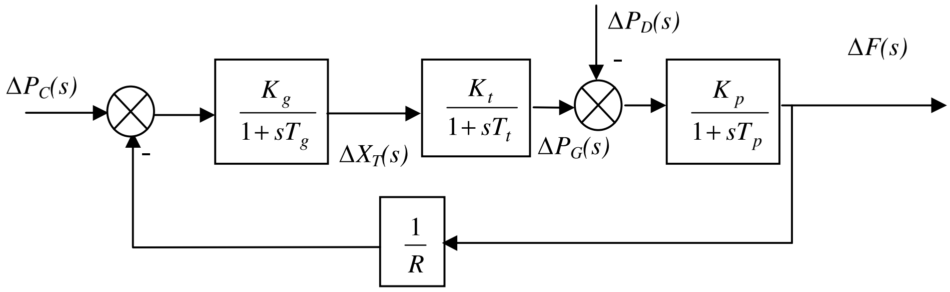


Fig 14.4

Where $\Delta P_G(s) = \Delta P_t(s)$

Steady state response-

The main aim of Automatic Load Frequency Control (ALFC) is to maintain desired output of a generator unit and controlling the frequency of the larger interconnection. The ALFC also helps to keep the net inter change of power between pool members at pre determined values. One of the basic objectives of the loop is to maintain constant frequency in spite of floating loads.

Output → Δf

Inputs → ΔP_C, ΔP_D

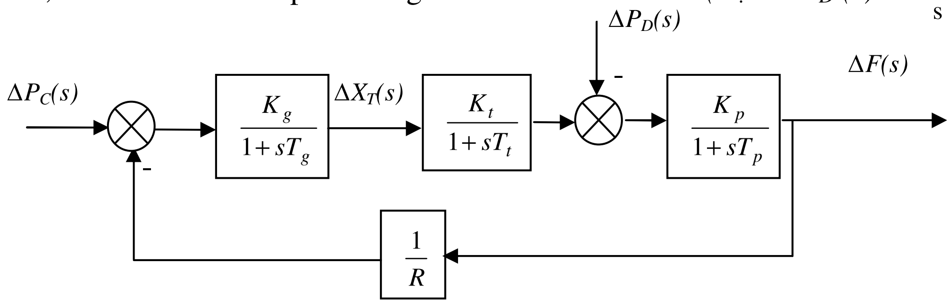
Where ΔP_C = change in speed changer setting, and

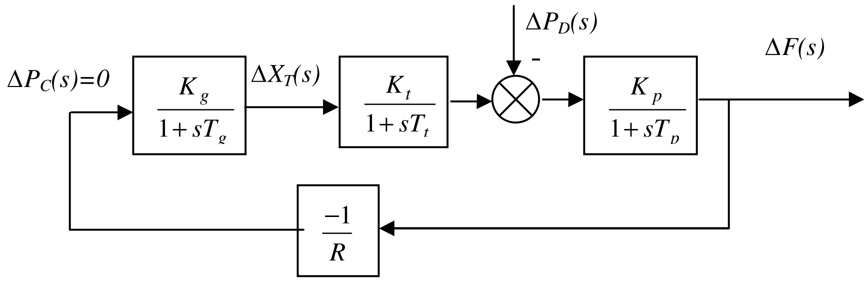
ΔP_D = change in load demand

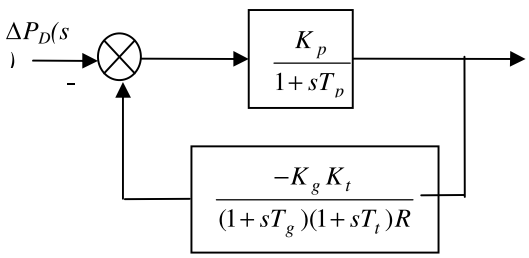
The speed changer has a fixed setting, i.e. $\Delta P_C = 0$ and the load demand changes. This called as free governor operation.

(i) Let us assume that he speed changer has a fixed setting $\Delta P_C\,(s) = 0$.

Assume that, there is a small step of change of load demand ΔP_D ($\because$ $\Delta P_D\,(s) = \frac{\Delta P_D}{s}$)







$$\Delta F(s) = \frac{-\frac{K_P}{1+sT_P}}{1+\frac{K_P.K_g.K_t}{(1+sT_P)(1+sT_g)(1+sT_t)R}}.\quad \Delta P_D(s) \qquad \text{but} \quad \Delta P_D\,(s) = \frac{\Delta P_D}{s}$$

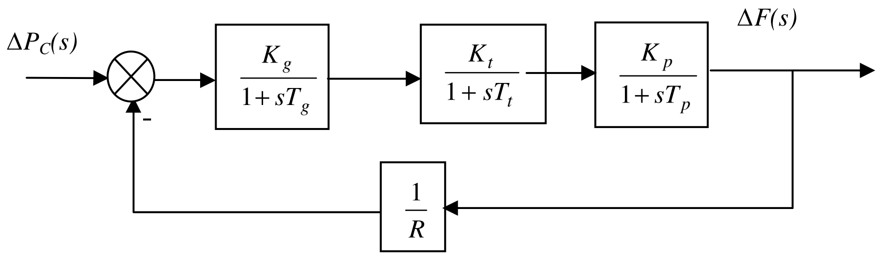
$$\Delta F(s) = \frac{-K_P}{(1+sT_P) + \dfrac{K_P.K_g.K_t/R}{(1+sT_g)(1+sT_t)}} \cdot \frac{\Delta P_D}{s}$$

$$\Delta f_{\text{steady state } \Delta PC = 0} = \;-= \left.\begin{matrix} s\Delta F(s) \\ s \to 0 \end{matrix}\right|_{\Delta P_C(s)} = \frac{K_P}{1+K_P K_g K_t/R}.\Delta P_D$$

K_t, K_P are fixed for turbine and power system. K_g is the speed governor gain. It can easily adjustable by varying the lengths of various links.

$K_P K_g \approx 1$ where $K_P = \dfrac{1}{D}$ and $D = \dfrac{\partial P_D}{\partial f}$ $(B = D)$, $\Delta f = \dfrac{1}{D+\dfrac{1}{R}}.\Delta P_D$

(ii) A step change in speed governor setting and the demand remains constant. i.e., $\Delta P_C(s) = \dfrac{\Delta P_C}{s}$ $\Delta P_D(s) = 0$



$$\Delta F(s)\big|_{\Delta P_D(s)=0} = \frac{K_P.K_g.K_t}{(1+sT_P)(1+sT_g)(1+sT_t) + \dfrac{K_P.K_g.K_t}{R}} \cdot \frac{\Delta P_C}{s}$$

$$\Delta f\big|_{steadystate\ \Delta P_D(s)=0} = \left.\begin{matrix} s\ \Delta F(s) \\ s \to 0 \end{matrix}\right|_{\Delta P_D(s)=0} = \frac{K_P.K_g.K_t}{1+\dfrac{K_P.K_g.K_t}{R}}.\Delta P_C$$

$K_t K_g \approx 1$, $K_P = \dfrac{1}{D}$, $\Delta f = \dfrac{1}{D+\dfrac{1}{R}}.\Delta P_C$

if the speed changer setting is changed by ΔP_C and also the load demand changes by ΔP_D, the steady state frequency change is obtained by superposition theorem. i.e.,

$$\Delta f = -\frac{1}{D+\dfrac{1}{R}}.\Delta P_D + \frac{1}{D+\dfrac{1}{R}}.\Delta P_C = \frac{1}{D+\dfrac{1}{R}}(.\Delta P_C - \Delta P_D)$$

$R \to$ Speed Regulation

$D \to$ Demanding coefficient

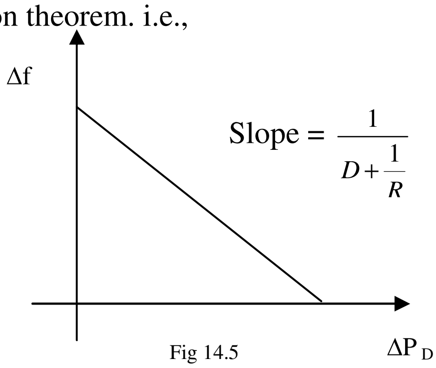


Fig 14.5

typical values of D=0.01 MW/Hz, R=3.

$$\Delta f \approx -R\,\Delta P_D$$

Dynamic Response

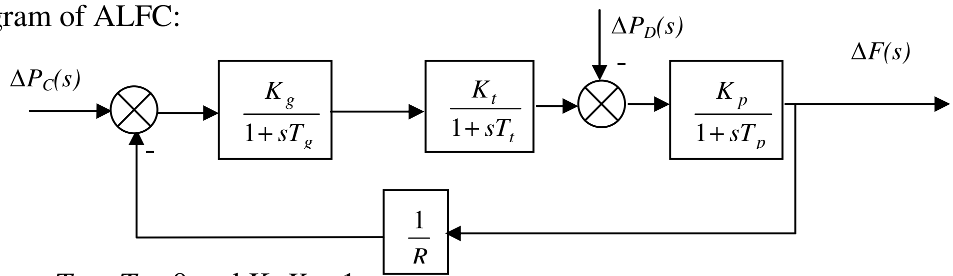
This gives the variation of frequency with respect to time for a given step change in load demand.

$\Delta F(s) \rightarrow$ Laplace transform of change in frequency

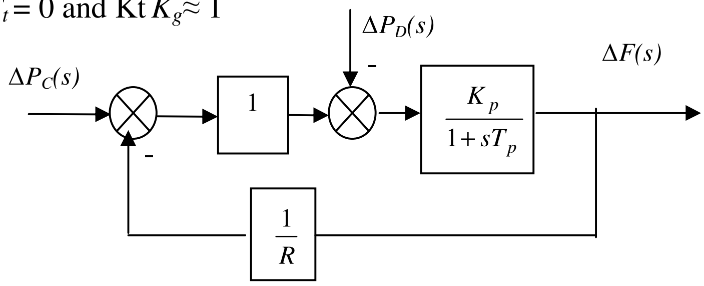
$f(t) \rightarrow$ Inverse Laplace transform of $\Delta F(s)$ time constant of load frequency control are follow the relation as

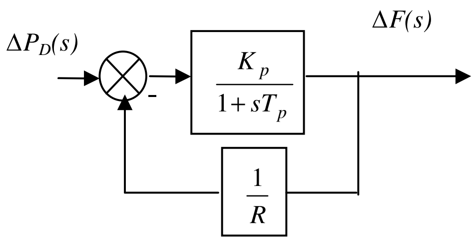
$$T_g < T_t < T_P$$

Block diagram of ALFC:



$T_g = T_t = 0$ and Kt $K_g \approx 1$





$$\Delta F(s)\Big|_{\Delta PC=0} = \frac{-\dfrac{K_p}{1+sT_p}}{1+\dfrac{K_p}{1+sT_p}\dfrac{1}{R}}\frac{\Delta P_D}{s}$$

$$= \frac{-\dfrac{K_p}{T_p}}{s[s+\dfrac{R+K_p}{RT_p}]}\Delta P_D$$

Taking partial fractions

$$= (\frac{A}{s}+\frac{B}{s+\dfrac{1}{T_p}(1+\dfrac{K_p}{R})})\Delta P_D$$

For s=0

$$A = \frac{-K_p}{1+\dfrac{K_p}{R}} = -\frac{RK_p}{R+K_p}$$

$For\ s = -\dfrac{(R+K_p)}{RT_p},$ $\qquad B = -\dfrac{RK_p}{R+K_p}$

$$\Delta F(s) = \frac{RK_p}{R+K_p}\left[\frac{-1}{s}+\frac{1}{s+\dfrac{1}{T_p}(1+\dfrac{K_p}{R})}\right]\Delta P_D$$

taking inverse LT

$$\Delta f(t) = L^{-1}\{\Delta F(s)\}$$

$$\Delta f(t) = -\frac{RK_p}{R+K_p}\left[1-e^{\frac{-1}{T_p}\left(1+\frac{K_p}{R}\right)t}\right]\Delta P_D$$

State space model for single area:

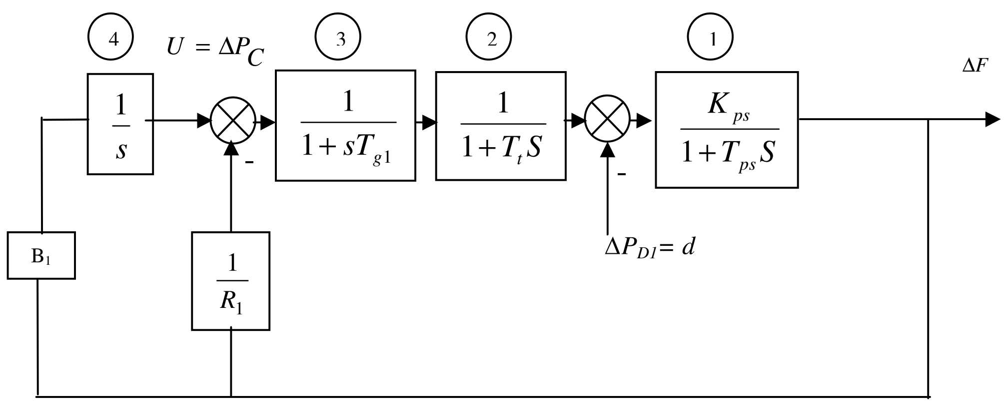


Fig 14.6

Let $U = \Delta P_C$.

be the states created by the linear combination of all our system state variables (x)

We define from the block diagram

$x_1 = \int \Delta f_1 dt$ $\qquad U = \Delta P_C$.

$x_2 = \Delta f$ $\qquad d = \Delta P_D$.

$x_3 = \Delta P_G$

$x_4 = \Delta P_{sg}$ (Speed governor)

From block (1)

$$x_2 = \left(\frac{K_{PS}}{1+T_{PS}}\right)(x_3 - d)$$

(14.1)

$$x_2 + x_2 T_{PS} S = K_{PS}(x_3 - d)$$

From equation 14.1

$$\dot{x}2 = \frac{1}{T_{PS}} x_2 + \frac{K_{PS}}{T_{PS}} x_3 - \frac{K_{PS}}{T_{PS}} d$$

(14.2)

From block (2)

$$x_3 = \left(\frac{1}{1+T_t S}\right)(x_4)$$

(14.3)

$$x_4 = x_3 + \dot{x}_3 T_t$$

From equation 14.2

$$\dot{x}_3 = \frac{-x_3}{T_t} + \frac{x_4}{T_t}$$

(14.4)

From block (3)

$$x_4 = (U - \frac{1}{R} x_2)(\frac{1}{1+T_{sg} S})$$

$$x_4 + x_4 T_{sg} S = (U - \frac{1}{R} x_2)$$

$$\dot{x}_4 = \frac{1}{T_{sg} R} x_2 - \frac{1}{T_{sg} R} x_4 + \frac{U}{T_{sg}}$$

(14.5)

From block (4)

$x_1 = \frac{1}{S} x_2$ $\qquad \dot{x}_1 = x_2$

MATRIX REPRESENTATION OF ALL STATE EQUATIONS

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & \frac{-1}{T_{PS}} & \frac{K_{PS}}{T_{PS}} & 0\\ 0 & 0 & \frac{-1}{T_t} & \frac{1}{T_t}\\ 0 & \frac{-1}{RT_{sg}} & 0 & \frac{-1}{T_{sg}}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 0\\ \frac{1}{T_{sg}}\end{bmatrix}(U) + \begin{bmatrix}0\\ \frac{K_{PS}}{T_{PS}}\\ 0\\ 0\end{bmatrix}(d)$$

It is in the form of

$$\dot{x} = Ax + Bu + FD \qquad (14.6)$$

U- control vector *D* - disturbance vector

x - state vector

The equation (6) into standard state space model form

i.e. $\dot{x} = Ax + Bu$ as follows

If x is having the sum of transient and steady state value and U is also having transient and steady state value

$$\left.\begin{aligned} x &= x^1 + x_{ss} \\ U &= U^1 + U_{ss}\end{aligned}\right\}$$

then substitute in equation (14.7)

$$\dot{x}^1 = Ax^1 + Ax_{ss} + BU^1 + BU_{ss} + FD \qquad (14.8)$$

If the sudden disturbance occurs and the system reached the steady state value

then $\dot{x} = 0$

$$Ax_{ss} + BU_{ss} + BU^1 + FD = 0$$

and transient term also zero under ss $\dot{x} = 0 \Rightarrow Ax_{ss} + BU_{ss} + FD = 0$

Now equation (7) is

$$\dot{x}^1 = Ax^1 + BU^1$$

MODULE IV:LOAD FREQUENCY CONTROL II

TWO AREA LOAD FREQUENCY CONTROL

Load frequency control of two-area system

In the single area case we could thus represent the frequency deviations by the single variable Δf. In the present case assume each area individual strong, and having them inter connected them with a weak tie-line

Therefore leads us to the assumption that the frequency deviation in the two areas can be represented by two variables Δf_1 and Δf_2 respectively.

An extended power system can be divided into a number of load frequency control areas inter connected by means of tie-lines. Consider a two area case connected by a single tie-line as shown in fig. Inter connected operation power system are inter connected for economy and continuity of power supply for the inter connected operation incremental efficiencies, fuel costs, water availabilities, spinning reserve allocation and area commitments are important considerations in preparing load dispatch schedules.

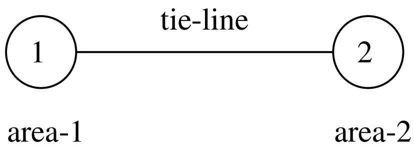


Fig 15.1 Inter connected two

15.1.1 Flat frequency control of inter connected system:

Let two generating stations are connected by a tie-line.

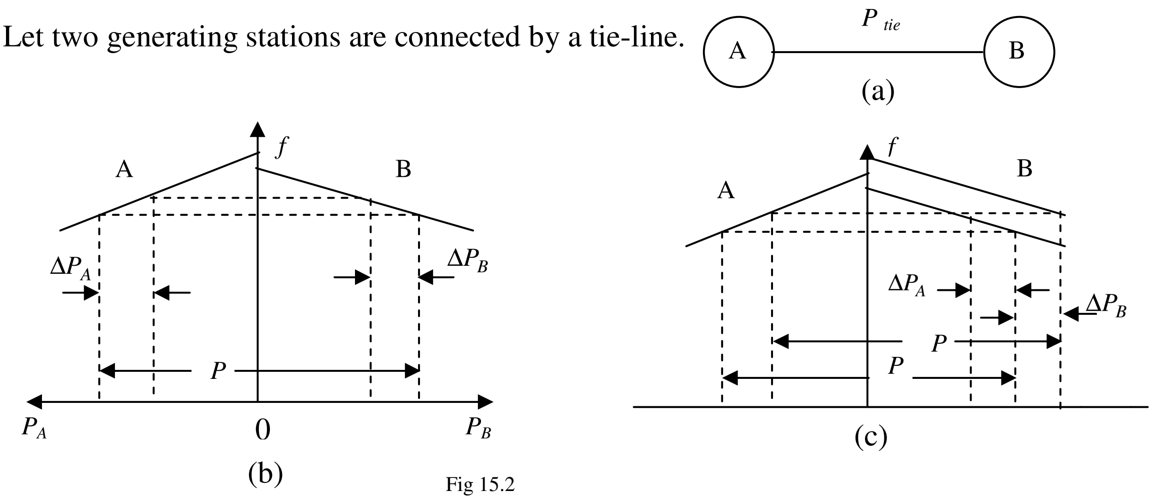


Fig 15.2

(a) Two inter connected stations.
(b) Uncontrolled system with load more on station B
(c) Frequency controller located at station B

If a load increment on station B, the kinetic energy of the generators reduces to absorb the same generations increases in both stations A and B and frequency will be less than normal at the end of the governor response period in fig15.2 (b).

The load increment will be shared partly by A and partly by B the tie-line power flow will change there by if a frequency controller is placed at B then it will shift the governor characteristics at B parallel to itself as shown in fig15.2(c) and the frequency will be restored to its normal value + reducing the change in generation in A to zero.

If the load increment comes on station A, then as before initially the generation in both A and B changes to absorbs the additional load while finally the additional load is absorbed by B only.

Station A absorbs none of its load changes in the steady state. It is possible that, in inter connected operation, a given station can be made to absorb the load changes occurring else where in the system so long as the controlling station has capability to absorb the change.

Same analysis can be applied to a two area system.

Assumptions in analysis:

Some assumptions are made in the analysis of the two area system

(1) The over all governing characteristics of the operating units in any area can be represented by linear curves of frequency verses generation.
(2) The governors in both the areas start acting simultaneously to changes in their respective areas.
(3) Secondly control device act after the initial governor response is over.

Tie-Line bias control:

With tie-line bias controllers located in both areas the control action is complete in all the cases. For the case where the tie-line bias controller in ‘A’ initiates action for local load changes, the response us shown in fig.

The response to controller action is similar for load changes in B when controller in B initiates regulating action.

Governor action is not changed till the area, where a load change has occurred, becomes effective. This avoids unnecessary change in generation, frequency or tie-line power the controller in the area, where load change occurs acts I such a manner that the area absorbs its own load change only a single shift is necessary to the governor characteristics to restore both frequency and tie-line power to normal. A smooth cooperative regulation thus achieved with a tie-line bias tie-line bias control scheme for the two area system.

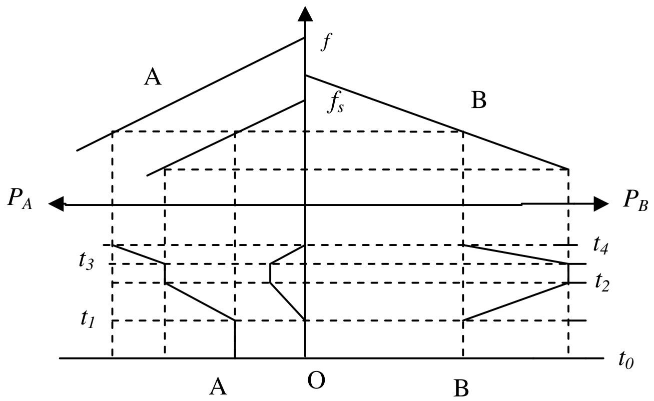


Fig 15.3

1 Block diagram representation for state space model for Two-area control

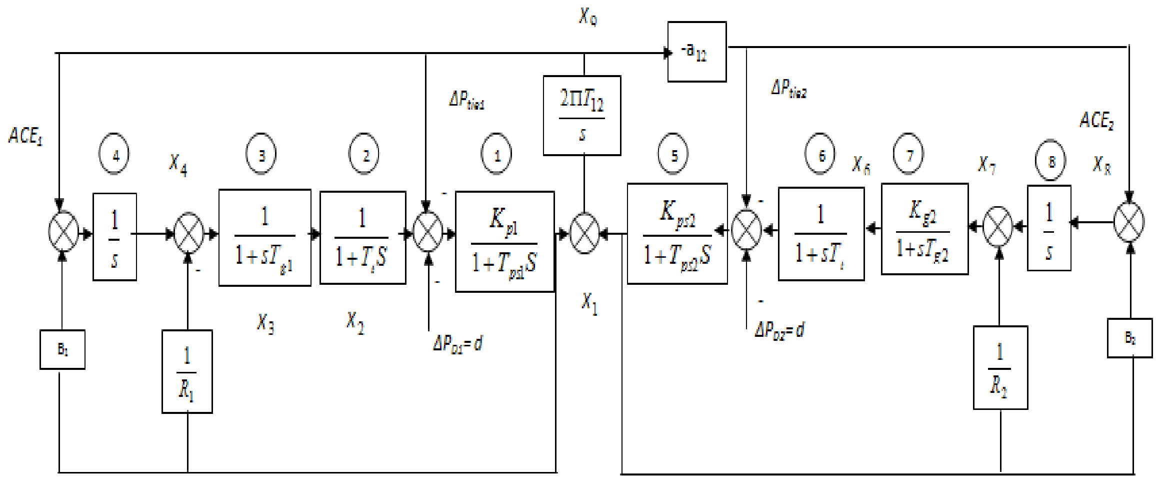


Fig 15.4

TWO AREA STATE SPACE MODEL REPRESENTATION

State space variables for two area load frequency control

$x_1 = \Delta t_1$,	$x_2 = \Delta PG_{1,}$	$x_3 = \Delta PG_{ST}$
$x_4 = \int ACE_1 dt$	$U_1 = \Delta P_4$	$d_1 = \Delta PD_1$
$x_5 = \Delta f_2$	$x_6 = \Delta PG_2$	$x_7 = \Delta PG_{S2}$
$x_8 = \int ACE_2 dt$	$U_2 = \Delta P_{c2}$	$d_2 = \Delta PD_2$

From block (1)

$$\dot{x}1 = \frac{-1}{T_{ps1}} x_1 + \frac{K_{ps1}}{T_{ps1}} x_2 - \frac{K_{ps1}}{T_{ps1}} x_9 - \frac{K_{ps1}}{T_{ps1}} d_1 \qquad (15.1)$$

From block (2)

$$\dot{x}2 = \frac{-1}{T_{t1}} x_2 + \frac{x_3}{T_{t1}} {}_2$$

(15.2)

From block (3)

$$\dot{x}3 = \frac{-1}{R_1 T_{sg1}} x_1 - \frac{1}{T_{sg1}} x_3 + \frac{1}{T_{sg1}} U_1$$

(15.3)

From block (4)

$$\dot{x}_4 = B_1 x_1 + x_9$$

(15.4)

From block (5)

$$x_5 = \frac{K_{ps2}}{1 + T_{ps2} S} [x_6 - d_2 + a_{12} x_9]$$

$$x_5 (1 + T_{ps2} S) = K_{ps2} [[x_6 - d_2 + a_{12} x_9]$$

$$\dot{x}5 = \frac{-1}{T_{ps2}} x_5 + \frac{K_{ps2}}{T_{ps2}} x_6 - \frac{K_{ps2}}{T_{ps2}} a_{12} x_9 - \frac{K_{ps2}}{T_{ps2}} d_2 \qquad (15.5)$$

From block (6)

$$\dot{x}_6 = \frac{-1}{T_{t2}} x_6 + \frac{1}{T_{t2}} x_7 \; x_7$$

(15.6)

From block (7)

$$\dot{x}7 = \frac{-1}{R_2 T_{sg2}} x_5 - \frac{1}{T_{sg2}} x_7 + \frac{1}{T_{sg2}} U_2$$

(15.7)

$$\dot{x}_8 = B_2 x_5 - a_{12} x_9$$

(15.8)

$$x_9 = \frac{2\Pi T_{12}}{S}(x_1 - x_5)$$

$$\dot{x}9 = 2\Pi T_{12}(x_1 - x_2)$$

In the matrix form above nine equations are written as

$$[\dot{x}\ ] = [A][x]^{-1}[B][U]+[F][D]$$

Where $[x] = [x_1, x_2, x_3, x_4 \ldots\ldots\ldots x_9]^T$ is state vector, $[\ U\ ] = [U_1, U_2]^T$ is control vector $D = [d_1,\ d_2]^T$ is a disturbance vector

$$A = \begin{bmatrix} \frac{-1}{T_{PS1}} & \frac{K_{ps1}}{T_{PS1}} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{K_{ps1}}{T_{PS1}} \\ 0 & \frac{-1}{Tt_1} & \frac{1}{Tt_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{RT_{sg1}} & 0 & \frac{-1}{T_{sg1}} & 0 & 0 & 0 & 0 & 0 & 0 \\ B_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{T_{PS2}} & \frac{K_{ps2}}{T_{PS2}} & 0 & 0 & a_{12}\frac{K_{ps2}}{T_{PS2}} \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{Tt_2} & \frac{1}{Tt_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{R_2 T_{sg2}} & 0 & \frac{-1}{T_{sg2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & B_2 & 0 & 0 & 0 & -a_{12} \\ 2\Pi T_{12} & 0 & 0 & 0 & -2\Pi T_{12} & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{T_{sg1}} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{T_{sg2}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad F = \begin{bmatrix} \frac{-K_{PS1}}{T_{PS1}} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{-K_{PS2}}{T_{PS2}} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

STEADY STATE ANALYSIS

UNCONTROLLED CASE

$\Delta P_{C1}(S) = \Delta P_{C2}(S) = 0$

(15.9)

In the case $\Delta P_{C1} = \Delta P_{C2} = 0$, the speed changer setting is fixed irrespective of load changer in area 1 and area 2.

Let a sudden change occurs in area 1 and area 2 is suddenly increased by incremental steps in ΔP_{D1} & ΔP_{D2}

ΔP_{D1} & ΔP_{D2} due to load change the frequencies drops by

Δf_1 & Δf_2 if the load increment in two areas are equal the frequencies drops are also equal (assume) if $\Delta f_1 = \Delta f_2 = \Delta f\ (ss)$ from the block diagram the incremental increase of generation is written as

$$\Delta P_{G1}(S) = \frac{-1}{R_1}\left(\frac{K_{t1}K_{sg1}}{(1+T_{sg1}s)(1+T_{sg1}s)}\Delta F_1(s)\right.$$

$$= \frac{-1}{R_1}\Delta F_1(s) \qquad \begin{bmatrix} \because K_{t1}K_{sg1} \approx 1 \\ T_{sg1} = T_{t1} = 0 \end{bmatrix}$$

$$= = \frac{-1}{R_1}\Delta F_{ss}(s) \qquad (15.10)$$

We have $\Delta P_{G1} - \Delta P_{D1(S)} - \Delta P_{tie1(S)} = \frac{2H}{f_0}\frac{d}{df}\Delta f_{SS}(S) + B_1\Delta f_{SS}(S)$

Since then $\Delta f_{SS}(S)$ is a steady state value

$$\frac{d}{dt}\Delta f_{SS}(S) \approx 0$$

For area 1

$$\Delta P_{G1} - \Delta P_{D1(S)} - \Delta P_{tie1(S)} = B_1\Delta f_{SS}(S) \qquad (15.11)$$

For area 2

$$\Delta P_{G2} - \Delta p_{D2(S)} - \Delta p_{tie2(S)} = B_2\Delta f_{SS}(S) \qquad (15.12)$$

Also we have

$$a_{12} = \left(\frac{rating1}{rating2}\right)$$

$$\Delta P_{tie1} = -a_{12}\Delta P_{tie2}$$

Using above (15.9) & (15.10) equations (15.11) & (15.12) are written as

$$-\frac{1}{R_1}\Delta f_{ss}(S) - \Delta P_{D1}(S) = B_1\Delta f_{1ss}(S) + \Delta P_{tie1}(S) \qquad (15\ 15)$$

$$-\frac{1}{R_2}\Delta f_{ss}(S) - \Delta P_{D2}(S) = B_2\Delta f_{2ss}(S) - a_{12}\Delta P_{tie1}(S) \qquad (15.14)$$

Subtracting (15.15) & (15.14)
Multiplying (15.15) with a_{12} and subtracting it from (15.14)
We will get

$$a_{12}(\frac{1}{R_1}\Delta f_1(S)+\Delta P_{D1}(S))+a_{12}[B_1\Delta f_1(S)]-B_2\Delta f_2(S)+[\frac{1}{R_2}\Delta f_2(S)+\Delta P_{D2}(S)]=0$$

$$\Delta f_1(S) = \Delta f_2(S) = \Delta f\ (S)$$

$$\Delta f(S.S)=\frac{-a_{12}\Delta P_{D1}+\Delta P_{D2}}{a_{12}(B_1+\frac{1}{R_1})+(B_2+\frac{1}{R_2})}$$

similarly from (15.15) & (15.14)
ΔP_{tie1} is obtained as
We have

$$-\frac{1}{R_1}\Delta f_1-\Delta P_{D1}=B_1\Delta f_1(S)+\Delta P_{tie1}$$

$$-\frac{1}{R_2}\Delta f_2-\Delta P_{D2}=B_2\Delta f_2(S)-a_{12}\Delta P_{tie1}$$

From (15.15)

$$\Delta P_{tie1}=-(B_1+\frac{1}{R_1})\Delta f_1-\Delta P_{D1}$$

(15.15)
From (15.14)

$$-a_{12}\Delta P_{tie1}=-(B_2+\frac{1}{R_2})\Delta f_2-\Delta P_{D2}$$

(15.16)
Multiplying (15.15) by $(B_2+\frac{1}{R})$ and (15.16) by $(B_1+\frac{1}{R_1})$
Then

$$\Delta P_{tie1}(B_2+\frac{1}{R_2})=-(B_1+\frac{1}{R_1})(B_2+\frac{1}{R_2})\Delta f_1-\Delta P_{D1}(B_2+\frac{1}{R_2}) \quad (15.17)$$

$$-a_{12}\Delta P_{tie1}(B_1+\frac{1}{R_1})=-(B_1+\frac{1}{R_1})(B_2+\frac{1}{R_2})\Delta f_1-\Delta P_{D2}(B_1+\frac{1}{R_1}) \quad (15.18)$$

Subtracting (15.17) & (15.18)
From (15.14)

$$-\frac{1}{R_2}\Delta f_2-\Delta P_{D2}=B_2\Delta f-a_{12}\Delta P_{tie1}$$

$$-a_{12}\Delta P_{tie1}=(B_2+\frac{1}{R_2})\Delta f-\Delta P_{D2}$$

$$\Delta P_{tie1}[(B_2+\frac{1}{R_2})+a_{12}(B_1+\frac{1}{R_1})]=(B_1+\frac{1}{R_1})\Delta P_{D2}-(B_2+\frac{1}{R_2})\Delta P_{D1}$$

$$\Delta P_{tie1}=\left(\frac{(B_1+\frac{1}{R_1})\Delta P_{D2}-(B_2+\frac{1}{R_2})\Delta P_{D1}}{(B_1+\frac{1}{R_1})a_{12}+(B_2+\frac{1}{R_2})}\right)$$

(15.19)

$$\Delta f(S.S) = \left(\frac{-\Delta P_{D2} + a_{12}\Delta P_{D1}}{(B_1 + \frac{1}{R_1})a_{12} + (B_2 + \frac{1}{R_2})} \right)$$

(15.20)

$R_1 = R_2 ; B_1 = B_2$

$\beta_1 = \beta_2 = \beta \quad ; a_{12} = 1$

Then ,Eqs.(15.19), (15.20) can be modified as

$$\Delta f(s.s) = \frac{\Delta P_{D1} + a_{12}\Delta P_{D2}}{\beta_2 + a_{12}\beta_1}$$

$$\Delta P_{tie1} = \frac{\beta_2 \Delta P_{D2} - \beta_2 \Delta P_{D1}}{\beta_2 + a_{12}\beta_1}$$

If two areas are identical then

$\beta_1 = \beta_2 = \beta \quad ; a_{12} = 1$

Eqs.(15.19), (15.20) can be modified as

$$\Delta f(s.s) = \frac{\Delta P_{D1} + \Delta P_{D2}}{2\beta}$$

$$\Delta P_{tie1} = \frac{\beta(\Delta P_{D1} - \Delta P_{D2})}{2\beta}$$

$$= \frac{(\Delta P_{D1} - \Delta P_{D2})}{2}$$

If step change in area 1 only

$\Delta P_{D2} = 0$

Then

$$\Delta f(s.s) = \frac{-\Delta P_{D1}}{2\beta}$$

$$\Delta P_{tie1} = \frac{-\Delta P_{D1}}{2}$$

From the above it is observed that with the interconnection and a step change in lead at area 1 only , the steady state change in frequency is 50% of steady state change of frequency of single area

$\Delta f(s.s) = \frac{-\Delta P_D}{2\beta}$ for single area ALF

$\Delta f(s.s) = \frac{-\Delta P_D}{\beta}$ for two area ALFC

Similarly from Δp_{tie} equation . It is observed that if a load change occurs in any area 50% is generated from area 1

CONTROLLED CASE

In the case of an isolated single-area system we used the integral control i.e. we let the speed changer command by a single obtained by first amplifying and then integrating the frequency error and which brought the steady state deviation in frequency to zero . so in the two area system our new control starategy should be such that both steady state frequency and tie-line deviations vanish due to change in either area. Area control error is to be defined to take core of frequency deviation and tie-line power deviation.

Under normal operating condition the Δf Δf (Δf_1 & Δf_2) and ΔP_{tie1} must be zero =

$$\Delta f = \frac{-\Delta P_{D2} + a_{12}\Delta P_{D1}}{\beta_2 + a_{12}\beta_2} \qquad \Delta p_{tie1} = \frac{-\Delta P_{D2} + a_{12}\Delta P_{D1}}{\beta_2 + a_{12}\beta_2}$$

But these two parameters are present under normal operating condition these are undesirable so here similar to single area we use PI controller and these two errors are brought to zero

Δf - Actual - normal

Here similar to single area the speed changed according the changes of Δf & ΔP_{tie1}

We define the ACE_1 (Area control error)

$$= B_1\Delta f_1 + \Delta P_{tie1}$$

$$ACE_1 = \mathrm{B}_2\Delta f_2 + \Delta P_{tie2}$$

Where B_1 & B_2 are frequency bias parameters

Now,

$$\Delta p_{c1} = -\mathrm{K}_{i1}\int ACE_1 dt$$

$$= -\mathrm{K}_{i1}\int B_1\Delta f_1 + \Delta P_{tie1}\ \mathrm{dt}$$

$$\Delta p_{c2} = -\mathrm{K}_{i2}\int ACE_2 dt$$

$$= -\mathrm{K}_{i2}\int B_2\Delta f_2 + \Delta P_{tie2}\ \mathrm{dt}$$

$\mathrm{k}_i \& k_2$ are integrator gains if ΔPD_1 & ΔPD_2 are applied simultaneously under steady state the outputs of all the block in two area ALFC constant under these conditions

$ACE_1(S.S) = B_1\Delta f_1(S.S) + \Delta P_{tie1}(S.S)$

$ACE_2(S.S) = B_2\Delta f_2(S.S) + \Delta P_{tie2}(S.S)a$

If $\Delta f_1(S.S)a = \Delta f_2(S.S) = \Delta f\ (S.S)$ and

$\Delta p_{tie2}\ (S.S) = -a_{12}\ \Delta p_{tie1}\ (S.S)$

We know under normal or steady state the two errors must be zero.

$B_1\Delta f_1(S.S) + \Delta P_{tie1}(S.S) = 0$

$-a_{12}\ \Delta p_{tie1}\ (S.S) + B_2\Delta f\ (S.S) = 0$

$$\begin{bmatrix} 1 & B_1 \\ -a_{12} & B_2 \end{bmatrix} \begin{bmatrix} \Delta P_{tie1}(S.S) \\ \Delta f(S.S) \end{bmatrix} = 0$$

Here under steady state Δf & ΔP_{tie} must be zero i.e. , each area has to supply its own load by itself and errors must be zero.

If ΔP_{tie} , Δf is zero then $|B_2 + a_{12}B_1| \neq 0$ the value of determinant will be zero otherwise if $B_1 = 0$ & $B_2 \neq 0$ Determinant $\neq 0$ vice versa

MODULE V:REACTIVE POWER CONTROL

REACTIVE POWER COMPENSATION

INTRODUCTION

The modern electric utility industry began in the 1880s. On September 4, 1882, the first commercial power station, located on Pearl Street in lower Manhattan, known as Pearl Street electricity generating station went into operation providing light and electricity power to customers in a one square mile area. This was a D.C power system built by Thomas Alva Edison, which was comprised of a steam driven DC generator connected through an electrical cable system at 110V and 59 customers spread over an approximate area with 1.5 km radius.

But now the electrical energy was generated, transmitted and distributed exclusively in the form of alternating current (AC). The main drawback of D.C generation is that it can neither be generated in bulk nor can be stepped up to higher levels for its efficient transmission purpose. Development of transformers changed the intact scenario of electrical power systems. Invention of induction motors (single phase and poly phase) by Nikola Tesla and massive increase in load demand has been called for restructuring of power systems. The majority of the loads in the present power system are of inductive in nature and when ever they are supplied from an A.C source, they draw a current which is lagging by the applied voltage at an angle and immediately power factor comes into picture.

Before discussing about the reactive power its worth revising about the concept of power factor since both of these share a very close relation with each other

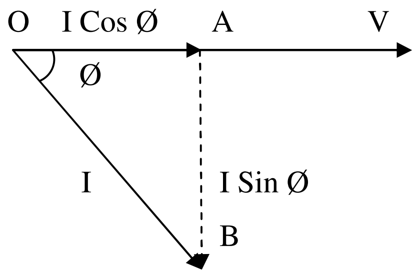
POWER FACTOR

The cosine of phase angle between the two electrical quantities such as voltage and current in an A.C circuit is known as ***power factor***.

For instance, consider a load supplied by a voltage V and drawing a current of magnitude I which lags the voltage by an angle Ø. The phasor diagram of the circuit is given in figure-1. The power factor of the circuit is given by,

Power Factor = Cos (Ø)

Figure 16.1



The load current I can be resolved into two components.

1. Horizontal component, I Cos Ø in phase with the applied voltage V

2. Vertical component, I Sin Ø, 90^0 out of phase with the applied voltage V

The horizontal component, I Cos Ø is known as active or wattful component and the vertical component, I Sin Ø is known as the reactive or wattles component. The reactive component is the measure of power factor. Greater the reactive component, larger the phase angle (Ø) between the applied voltage and the current drawn by the load, hence the power factor Cos Ø is low. Therefore a circuit having small reactive current I Sin Ø will have high power factor and vice-versa.

Power factor is a characteristic of alternating current (AC) circuits. Its value is always between (0.0) and (1.0), the higher the number the greater or better the power factor. If a purely resistive load is connected to a power supply, current and voltage will be in phase with each other, and the angle Ø equals to zero, then consequently power factor will be unity and the electrical energy flows in a single direction across the network in each cycle.

Inductive loads such as transformers and motors (any type of wound coil) generate reactive power with current waveform lagging the voltage. Capacitive loads such as capacitor banks or buried cable generate reactive power with current phase leading the voltage. Both types of loads will absorb energy during part of the AC cycle, which is stored in the device's magnetic or electric field, only to return this energy back to the source during the rest of the cycle.

The power factor concept can also be analyzed in terms of power drawn by the load, from the power triangle shown in the figure - 16.2

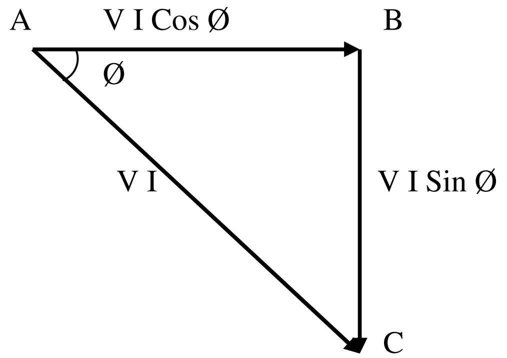


Figure-16.2 **Power triangle**

The right angled triangle ABC shown in figure – 16.2 is known as power triangle and is directly drawn from the triangle OAB in the phasor diagram from figure -16.1. Each side of the current triangle OAB of figure -16.1 is multiplied by voltage V, to get the power triangle ABC shown in figure -16.2.

From the power triangle ABC, we can obtain some basic definitions

- **AB =** V I Cos Ø**,** which represents **Active Power** (also called ***Actual Power*** or ***Working Power*** or ***Real Power***) and is measured in watts or kilowatts i.e., W or kW.
- **BC =** V I Sin Ø**,** which represents **Reactive Power** and is measured in Volt Ampere Reactive i.e.,VAR's or Kilo Volt Ampere Reactive KVAR's.
- **AC =** V I**,** which represents **Apparent Power** and is measured in Volt Amperes or Kilo Volt Amperes i.e.,VAR's or KVAR's. It is the "vectorial summation" of **KVAR** and **KW**.

The power factor can also be defined in terms of power by inferring some points from the power triangle. The apparent power VI has two components. One is V I Cos Ø i.e., **Active Power** and the other one is V I Sin Ø i.e., **Reactive Power** at right angles to each other.

From the power triangle which is a right angled one we can write,

$$AC^2 = AB^2 + BC^2$$

$$(\text{Apparent power})^2 = (\text{Active power})^2 + (\text{Reactive power})^2$$

$$(\text{kVA})^2 = (\text{kW})^2 + (\text{kVAR})^2$$

Power factor, Cos Ø = AB / AC = Active power / Apparent power = kW / kVA

So the power factor can also be defined as the ratio of Active Power to Apparent Power. It is clear from the power triangle that the lagging reactive power due to the lagging current is responsible for the low power factor. The smaller the reactive power, higher the power factor and vice versa.

REACTIVE POWER

In a purely resistive AC circuit, voltage and current waveforms are in phase, reverse their polarity at the same instant in each cycle (illustrated in figure - 16.3). When reactive loads are present (such as capacitors or inductors), energy is stored in the form of electric or magnetic fields respectively in the loads during part of the AC cycle and results in a time difference between the current and voltage waveforms (for a capacitor, current leads voltage; for an inductor, current lags voltage). This stored energy returns to the source during rest of the cycle and is not available to do work at the load. This energy continuously flowing back and forth (to and fro) is known as reactive power while the active power flows from one point of the network to another. For a complete cycle, the net reactive power flow is zero as the amount of energy flowing in one direction for half a cycle is equal to the amount of energy flowing in the opposite direction in the next half of the cycle.

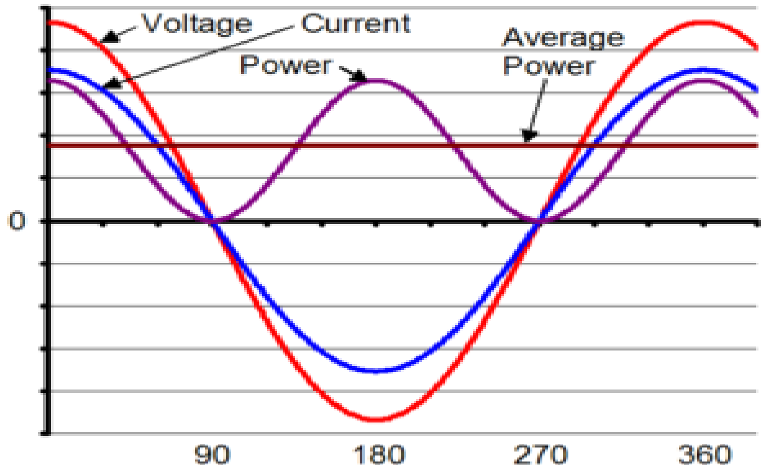


Figure 16.3

SOURCES OF REACTIVE POWER

The chief sources of reactive power are given below.

1. Overhead Lines and Under ground Cables: When current flows through a line it produces a magnetic field which absorbs reactive power given by (I^2 X) where I is the current flowing through the line and X is the reactance of the line in ohms per phase. A lightly loaded overhead line is a net generator of reactive power whereas a heavily loaded line is a net absorber of reactive power. But underground cables, have a small inductance and relatively large capacitances due to closeness, large size of conductors and high relative permittivity dielectric material used and so generate reactive power.

2. Transformers and Motors: Transformers produce magnetic fields and therefore absorb reactive power. Inductive loads such as motors (any type of wound coil) absorb reactive power.

CAUSES OF LOW POWER FACTOR

Circuits containing purely resistive loads such as filament lamps, strip heaters, cooking stoves, etc., operate at unity power factor. When the power factor is 1, all the energy supplied by the source is consumed by the load. Circuits containing inductive or capacitive elements usually have a power factor below 1.0. When power factor is equal to 0, the energy flow is entirely reactive, and stored energy in the load returns to the source on each cycle. Good power factor is considered to be greater than 0.85 or 85%.

1. Most of the A.C motors (any type of wound coil) are of induction type i.e., single phase or three phase induction motors consume reactive power with current waveform lagging the voltage and consequently operate at a poor power factor.
2. Lightening loads such as Arc lamps, electric discharge lamps, etc., operate at a low lagging power factor.
3. Industrial heating furnaces such as Arc furnaces usually operate on the principle of striking of arc operate at a low lagging power factor.
4. Normal power factor ballasts (NPF) typically have a value of (0.4) - (0.6). Ballasts with a power factor greater than (0.9) are considered to be high power factor ballasts (HPF).

DISADVANTAGES OF LOW POWER FACTOR

Power factor plays a vital role in A.C circuits since power consumed depends upon the operating power factor.

As it is evident that the Real power $P = V\ I \cos Ø$

$$I = P / V \cos Ø$$

It is clear from the above expression that, for a fixed power and voltage the load current is inversely proportional to the power factor. Lower the power factor, higher will be the load current and vice versa.

Low power factor results in the following disadvantages:

1. Increase in copper losses

When a load has a power factor lower than 1, more current is required to deliver the same amount of useful energy. Copper losses ($I^2 R$ losses) increase with increase in current. This results in poor efficiency with increase in losses.

2. Large conductor size

To transmit a given power at a constant voltage, the conductor will have to carry relatively more current at low power factor. This obliges greater conductor size i.e. the cross-sectional area of the transmission lines, cables and motor conductors has to be designed based on the increased current. This involves more cost. This case is illustrated in a numerical example-1 for clear understanding.

3. Large kVA rating

The electric machinery for example alternators, transformers, motors, switchgear etc., are always rated in kVA. And kVA = kW / Cos Ø. From this relation it is clear that kVA rating of the equipment is inversely proportional to the power factor. Lower the power factor, larger will be the kVA rating. Therefore at low power factor, the kVA rating of the equipment should be made more, making the equipment size larger and expensive.

4. Poor voltage regulation

The large current at low lagging power factor results in greater voltage drops (IZ) in power system components like alternators, transformers, transmission and distribution lines etc. This increased voltage drop consequently results in poor voltage regulation and decreased voltage at the receiving end, which impairs the performance of the consumer loads. In order to maintain receiving end or consumer end voltages within permissible limits additional equipment like voltage regulators, booster transformers, on load tap changing transformers etc. are required which are expensive in nature.

5. Reduced handling capacity

Since the increase in the reactive component of current prevents the full utilization of installed capacity, the lagging power factor reduces the handling capacity of all the elements of the system.

6. Increases generation and transmission costs

The significance of power factor lies in the fact that utility companies supply customers with volt-amperes, but bill them for watts. Power factors below 1.0 require a utility to generate more than the minimum volt-amperes necessary to supply the Working Power i.e. the Real power (watts). This increases generation and transmission costs. Utilities may charge additional costs or penalize the customers (not domestic loads but large consumers like industries etc.,) who have a power factor below some limit. This case is illustrated in a numerical example-2 for clear understanding.

LOAD COMPENSATION

In order to have an economic AC power transmission, the given transmission line should be able to transmit as maximum power as possible. There are two main factors affecting the maximum power transmission.

1. Synchronous machines should be remained in synchronism by maintaining power system stability
2. Voltages should be maintained very near to their rated values inspite of the system over loading or under loading conditions

The development of compensation techniques is making AC power Transmission technically and economically competitive.

Load compensation is the management of reactive power to improve the quality of supply in AC power systems. Load compensation is termed when the compensation equipment is usually being installed at the load end for supplying reactive power rather than supplying it from a distant generating station. The main objectives of load compensation are power factor correction and improvement of voltage regulation.

Power factor correction: Most of the power system loads operate at lagging power factors which implies that they consume reactive power. Therefore the load current drawn by these inductive loads which operate at poor lagging power factors tends to be larger than required. This excess load current is not economically justifiable as far as a consumer tariff is concerned. The supply utilities cannot supply reactive power from their generators since it is also not economically feasible. Therefore power factor correction plays a vital role in improving the power factor by generating required reactive power locally (at the loads) than supplying it from a remote generating station.

Improvement of voltage regulation: Modern power systems are sensitive for abnormal voltages and voltage fluctuations even for shorter durations. The fundamental requirement of AC power transmission is to maintain the system voltage profile at rated voltage levels or within a prescribed voltage range ($\pm$ 5% of supply voltage). All loads vary their demand for reactive power and this variation in reactive power causes corresponding variations in voltage which is not desirable. Therefore compensating devices which compensates the reactive power at the load point directly has a vital role to play in maintaining supply voltages within the defined limits.

SPECIFICATIONS OF A LOAD COMPENSATOR

The factors to be considered when specifying a load compensator are given below

1. Maximum reactive power requirement
2. Rated voltage, limits of voltage constraints and accuracy of voltage regulator required
3. System frequency and its variation
4. Overload rating and withstand capacity
5. Time of response after a particular disturbance
6. Protection and its coordination with other protective systems and other additional control requirements
7. Maximum harmonic distortion due to the installation of load compensator
8. Effect of unbalanced load and supply voltages on the performance characteristics of load compensator
9. Provisions for future expansion and rearrangements
10. Reliability and redundancy of the components of the load compensator
11. Some environmental factors like: noise, temperature, humidity, pollution, wind etc.,
12. Other requirements like grounding, cabling, enclosing, indoor/outdoor installation and cooling system

UNCOMPENSATED TRANSMISSION LINES

The study of uncompensated transmission lines starts from the fundamental transmission line equation. Generally the transmission line is characterized by four distributed circuit parameters such as its series resistance ‘r’, inductance ‘l’, shunt conductance ‘g’ and Capacitance ‘c’, usually per mile values. These parameters depend upon the conductor size, type, and spacing, height above the ground, frequency and temperature which are the functions of line design. The parameters give different values for positive sequence and zero sequence currents and also vary according to the number of nearby parallel lines. The actual characteristic behavior of the line is dominated by the series inductance and shunt capacitance. Series resistance is a secondary and has a separate importance in determining power losses. Shunt conductance is ignored and positive sequence nominal values are assumed.

The general wave equation governing the propagation of energy along a transmission line is given as

$$\frac{\partial^2 V}{\partial x^2} = \gamma^2 V \quad \text{———} \quad (1)$$

$$\frac{\partial^2 I}{\partial x^2} = \gamma^2 I \quad \text{———} \quad (2)$$

$$\frac{\partial V}{\partial x} = I Z \quad \text{———} \quad (3)$$

$$\text{where } \gamma^2 = (r + j\omega l)(g + j\omega c) \longrightarrow (4)$$

$$Z = (r + j\omega l) \quad , \quad Y = (g + j\omega c)$$

The above equations describe the variation of the voltage and current along the line and 'x' is the distance measured from any convenient reference point along the line. Let 'l' be the length of the line and the line is assumed as lossless (r=0, g=0), the general solution of the above equations are :

$$V(x) = V_r \cos\beta(l-x) + j Z_o I_r \sin\beta(l-x) \quad \text{—} \quad (5)$$

$$I(x) = j\left[\frac{V_r}{Z_o}\right] \sin\beta(l-x) + I_r \cos\beta(l-x) \quad \text{—} \quad (6)$$

The value of β can be determined from a propagation constant r by putting r = g= 0.

$$\gamma = \alpha + j\beta \text{ , for lossless system } \alpha = 0$$

$$\because \gamma^2 = (r + j\omega l)(g + j\omega c) = -\omega^2 l c \quad \text{——} \quad (7)$$

$$\therefore \gamma = j\beta$$

From the above equations

$$\beta = \omega\sqrt{LC} \quad \text{——} \quad (8)$$

$$\text{where } \omega = 2\pi f$$

$$\beta = \frac{2\pi f}{u} = \frac{2\pi}{\lambda} \quad \text{where } \lambda \text{ is wave length} \quad \text{—} \quad (9)$$

From the above equations we can understand these following things:

1. The real and imaginary parts of voltage and currents are sinusoidally varying along the line and they are said to form a standing wave.

2. The quantity √LC is called "Shilling Fraction" is the propagation velocity of electromagnetic effects along the line. For overhead transmission lines its value is less than the velocity of light $\mu = 3 \times 10^8$ m/sec.
3. The quantity β is the wave number, the number of complete waves per unit length of line and the quantity βl is the electrical length in radians.

The constant Z_0 in equation (5) is the surge impedance which is also called as characteristic impedance

$$Z_0 = \sqrt{(L/C)} \qquad \text{\_\_\_\_\_\_\_\_\_\_} \qquad (10)$$

Surge impedance is defined as the apparent impedance of an infinitely long line, which is the ratio of voltage to current at any point long it. An infinite line can be simulated by termination any line by its surge impedance.

Therefore if,

$V_r/I_r = Z_0$ then from equations (5) & (6)

$$Z(x) = \frac{V(x)}{I(x)} = \frac{Z_0 I_r \left[\cos\beta(l-x) + j\sin\beta(l-x)\right]}{I_r\left[\cos\beta(l-x) + j\sin\beta(l-x)\right]}$$

$$= Z_0 \quad \text{———} \quad (11) \text{ which is independent of 'x'.}$$

$$V(x) = V_r\left[\cos\beta(l-x) + j\sin\beta(l-x)\right] = V_r\, e^{j\beta[l-x]} \quad \text{—} (12)$$

$$I(x) = I_r\left[\cos\beta(l-x) + j\sin\beta(l-x)\right] = I_r\, e^{j\beta(l-x)} \quad \text{—} (13)$$

When the line is terminated through Z_0, the amplitude of both V & I have constant values all long the line. Therefore the line is said to have a "Flat Voltage profile". A line in this condition is said to be naturally loaded. The natural load or surge impedance load SIL is

$$P_0 = \frac{V_0^2}{Z_0}$$

Where, P_0 is the per phase value of surge – impedance power
V_0 is the nominal line – to – line voltage
Z_0 is the surge impedance (real Number)

The natural load of the uncompensated line increases with the square of the voltage.

At natural load the power factor is unity at all points along the line, including the ends. This can be deduced from equation (ii). It means that at the natural load no reactive power has to be generated or absorbed at either end. The reactive power generated in the shunt capacitance of the line is completely absorbed by the series inductance. Let the per unit length generated by the shunt capacitance is $V^2 b = V^2 \omega C$, and the reactive power per unit length absorbed by the series inductance is $I^2 \omega l$. For reactive power balance of the line,

$$V^2 \omega C = I^2 \omega l$$

$$\frac{V}{I} = \sqrt{\frac{l}{C}} = Z_0$$

Therefore the reactive power balance is achieved at the natural loading with $P_0 = V^2/Z_0$

<u>Uncompensated line open circuited at the receiving end</u>

Let us assume that the line is open circuited at the receiving end i.e., $I_r = 0$ therefore,

$$V(x) = V_r \cos\beta(l-x) \quad \text{——} \quad (15)$$

and $$I(x) = j\left[\frac{V_r}{Z_0}\right]\sin\beta(l-x) \quad \text{——} \quad (16)$$

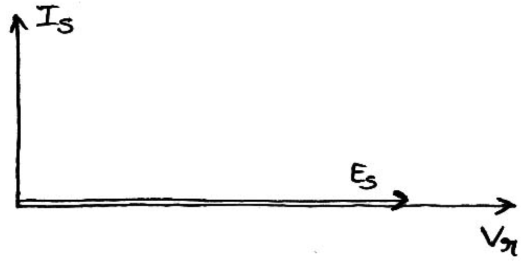
With $x = 0$; the voltage and current at the sending end are given by

$$V_s = V_r \cos\theta \;; = V_r \cos(\beta l) \quad \text{——} \quad (17)$$

$$I_s = j\left[\frac{V_r}{Z_0}\right]\sin\theta = j\left[\frac{V_s}{Z_0}\right]\tan\theta \quad \text{——} \quad (18)$$

$$= j\left[\frac{V_s}{Z_0}\right]\sin\beta l = j\left[V_s/Z_0\right]\tan\beta l$$

The voltages V_s and V_r are in phase with each other which implies that there is no power transfer the phasor diagram is shown



The line voltage profile expressed by equation (16) can be written more conveniently in terms of V_s.

$$V(x) = V_s \frac{\cos\beta(l-x)}{\cos\theta} \quad \text{——} \quad (19)$$

Similarly the current profile is given by

$$I(x) = j\frac{V_s}{Z_0}\frac{\sin\beta(l-x)}{\cos\theta} \quad \text{——} \quad (20)$$

The symmetrical line at no-load

"A line having identical synchronous machines connected at both ends is called symmetrical line".

The Symmetrical line at no-load is the line which is open circuited and energized from one end with identical synchronous machines at both ends, but no power transfer. If the terminal voltages are controlled to have the same magnitude, then $V_s = V_r$ from equations (2) and (3)

$$V(x) = V_r \cos\beta(l-x) + j Z_0 I_r \sin\beta(l-x)$$

and

$$I(x) = j\left[\frac{V_r}{Z_0}\right]\sin\beta(l-x) + I_r \cos\beta(l-x)$$

With $x = 0$

$$V_s = V_r \cos\theta + j Z_0 I_r \sin\theta \qquad (21)$$

$$I_s = j\left[\frac{V_r}{Z_0}\right]\sin\theta + I_r \cos\theta \qquad (22)$$

With no power transfer the electrical conditions are the same at both ends. Therefore by symmetry

$$I_s = -I_r \qquad (23)$$

From equation (22)

$$-I_r = j\frac{V_r}{Z_0}\frac{\sin\theta}{1+\cos\theta} = j\frac{V_r}{Z_0}\tan\theta/2 \qquad (24)$$

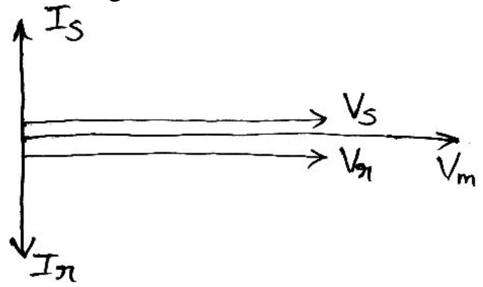
Substituting this value for I_r in equation (21)

gives $\quad V_s = V_r \qquad (25)$

and therefore $\quad I_s = j\dfrac{V_s}{Z_0}\tan\theta/2 \qquad (26)$

Equation (25) shows that V_r and V_s are in phase with each other with the fact that there is no power transfer. The current at each end is line – charging current.

The phasor diagram is shown in the figure



By Symmetry, the mid – point current is zero and the mid point voltage is therefore equal to the open circuit voltage of a line having half the total length.

$$V_m = \frac{V_s}{\cos(\theta/2)} \quad \text{———} \quad (27)$$

The voltage and current profiles for the Symmetrical line at no load can be derived from equation

$$V(x) = \frac{V_s \cos\beta(l-x)}{\cos\theta}$$

$$I(x) = j\frac{V_s}{Z_0}\frac{\sin\beta(l-x)}{\cos\theta}$$

with l replaced by $l/2$

$$V(x) = \frac{V_s \cos\beta(l/2 - x)}{\cos\theta/2} \quad \text{——} \quad (28)$$

and $$I(x) = j\frac{V_s}{Z_0}\frac{\sin\beta(l/2 - x)}{\cos\theta/2} \quad \text{——} \quad (29)$$

for $x \leq l/2$

For the other half of the line i.e, $l/2 \leq x \leq l$

$$V(x) = V(l-x) \quad \text{——} \quad (30)$$

and

$$I(x) = -I(l-x) \quad \text{——} \quad (31)$$

When the line is to be energized, the generator has to supply leading Vars to the line and with that $I_r = 0$ the charging reactive power at the sending end is given by

$$Q_s = I_m(V_s I_s^*) \quad \text{———} \quad (32)$$

Using the value of I_s from equation

$$I_s = j\frac{V_s}{Z_0}\tan\beta l \quad \text{———} \quad (33)$$

We have

$$Q_s = -P_c \operatorname{Tan}\beta l = -P_c \operatorname{Tan}\theta \quad \text{——} \quad (34)$$

The Charging current leads the line terminal voltage by 90^0 and flows in generators.

The reactive power absorption capability of synchronous generators is limited for two reasons.

1. It increases the heating of the stator core
2. The reduced field current reduces the internal emf of the generators and this impairs stability.

The absorption limit is typically not more than 0.45 pu of the MVA rating.

Therefore, the under excited operation of generators can set a more inflexible limit to the maximum length of an Uncompensated line than the open circuit voltage rise.

The uncompensated line under load

A load P-jQ at the receiving end of a transmission line draws the current

$$I_r = \frac{P - jQ}{V_r} \quad \text{———} \quad (35)$$

From equation (2) with x = 0 if the line is assumed loss less the sending end and receiving end voltages are related by

$$V_s = V_r \cos\theta + j Z_0 \sin\theta \, \frac{P - jQ}{V_r} \quad \text{———} \quad (36)$$

For a fixed V_s, the above quadratic equation can be solved for V. The solution shows how the receiving end voltage varies with the load and its power factor and with the line length.

The load power factor influences strongly on the receiving end voltage. Loads operating at lagging power factor, leading power factor (or) with unity tend to reduce the receiving end voltage as the load 'P' increases. Leading power factor loads generate reactive power which supplements the line charging reactive power and support the line voltage.

Maximum Power and Stability Considerations

Consider a symmetrical loss less line having a load of P + jQ connected at the receiving end. The terminal voltage of such a line is related by:

$$V_s = V_r \cos\theta + j Z_0 \frac{P - jQ}{V_r} \sin\theta \quad \text{———} \quad (37)$$

Assuming the load to be synchronous and taking V_r as reference, then

$$V_s = |V_s| (\cos\delta + j \sin\delta) \quad \text{———} \quad (38)$$

Where δ is the phase angle between V_s and V_r.
δ is called the load angle or the transmission angle. Equating the real and imaginary parts of the above equations (37) and (38)

$$V_s \cos\delta = V_r \cos\theta + Z_0 \frac{Q}{V_r} \sin\theta \quad \text{——} (39)$$

$$V_s \sin\delta = Z_0 \frac{P}{V_r} \sin\theta \quad \text{——} (40)$$

By rearranging we get,

$$P = \frac{V_s V_r}{Z_0 \sin\beta l} \sin\delta \quad \text{——} (41)$$

If the length of the line is electrically short,

$$\sin\beta l = \beta l = \omega\sqrt{LC}\cdot l$$

$$\text{and} \quad Z_0 \sin\beta l = \sqrt{\frac{L}{C}}\ \omega\sqrt{LC}\cdot l = l L \omega = X_0$$

Where, X_0 is the total reactance of the line.
Equation (41) therefore reduces to a very well known equation

$$P = \frac{V_s V_r}{X} \sin\delta \quad \text{——} (42)$$

Let $V_s = V_r$, then

$$P = \frac{V_r^2}{Z_0 \sin\beta l} \sin\delta = \frac{P_c}{\sin\beta l} \sin\delta. \quad \text{——} (43)$$

P is maximum when $\delta = 90^0$, which depends upon the length of the line where longer the length of the line smaller is the maximum value of P.

Let us consider a line with sending end source equivalent to a voltage source with open circuit voltage V_0 and impedance (R + jX) and a variable resistive load (R_1) Connected at the receiving end which is operating at unity power factor Now, we have to find out the value of R_1 for which the power transfer is maximum

$$\text{short circuit current } I_{sc} = \frac{V_0}{Z} = \frac{V_0}{R+jX}$$

$$\text{short circuit p.f. } \cos\phi_{sc} = \frac{R}{Z}$$

$$\text{The load current } I = \frac{V_0}{(R+R_1)+jX}$$

$$\therefore \text{power delivered} = \frac{V_0^2}{(R+R_1)^2+X^2} \times R_1 = P \quad \text{——} (44)$$

According maxima theorem, the power delivered will be maximum when $dP/dR_1 = 0$, Differentiation the above power expression (44) with respect to R_1 and equating it to Zero we get,

$$\left[(R+R_1)^2 + X^2\right] - R_1 \times 2(R+R_1) = 0$$

$$R_1^2 + R^2 + 2RR_1 + X^2 - 2RR_1 - 2R_1^2 = 0$$

$$R^2 + X^2 - R_1^2 = 0$$

$$R^2 + X^2 = R_1^2$$

$$\therefore \quad R_1 = Z$$

Substituting the above condition in the above expression of power (44) we get maximum power

$$\therefore \quad P_{max} = \frac{V_0^2 Z}{(R+Z)^2 + X^2} = \frac{V_0 I_{sc} Z^2}{R^2 + Z^2 + 2RZ + X^2} \quad [\because V_0 = I_{sc} Z]$$

$$= \frac{V_0 I_{sc} Z^2}{R_1^2 + Z^2 + 2RZ} \quad \left[\because R^2 + X^2 = R_1^2\right]$$

$$= \frac{V_0 \, I_{SC} \, Z^2}{Z^2 + Z^2 + 2RZ} \qquad \left[\text{Substituting the condition for max. power } (R_1 = Z) \right]$$

$$= \frac{V_0 \, I_{SC} \, Z^2}{2Z^2 + 2RZ}$$

$$= \frac{V_0 \, I_{SC} \, Z^2}{2Z\left[Z + R\right]} = \frac{V_0 \, I_{SC} \, Z}{2(Z + R]}$$

From power triangle $\cos\phi_{SC} = R/Z$

$$1 + \cos\phi_{SC} = 1 + \frac{R}{Z} = \frac{Z+R}{Z}$$

$$\therefore \frac{Z}{Z+R} = \frac{1}{1+\cos\phi_{SC}}$$

$$P_{max} = \frac{V_0 \, I_{SC}}{2\left[1 + \cos\phi_{SC}\right]} \quad \text{——— (45)}$$

Let V_0 be the open circuit voltage, V_r when $I_r = 0$ due to open circuited condition

Now from equation $V(x) = V_r \cos(l - x)\beta + j Z_0 I_r \sin \beta(l - x)$ we get,

$$V_s = V_0 \cos\beta l$$

$$\therefore V_0 = \frac{V_s}{\cos\beta l} \quad \text{——— (46)}$$

Let short circuit current I_{SC} is the value of I_r when $V_r = 0$ due to short circuited condition.
From basic equation

$$V(x) = V_r \cos(l-x)\beta + j Z_0 I_r \sin\beta(l-x)$$

we get,

$$V_s = j I_{sc} Z_0 \sin\beta l$$

$$I_{sc} = \frac{V_s}{j Z_0 \sin\beta l} \quad \text{------} (47)$$

Assuming the line to be lossless, $\cos\phi_{sc} = 0$

$$\therefore P_{max} = \frac{V_0 I_{sc}}{2} = \frac{V_s}{2\cos\beta l} \cdot \frac{V_s}{Z_0 \sin\beta l} = \frac{V_s^2}{Z_0 \sin 2\beta l} \rightarrow (48)$$

The above equation represents loci of maximum power for different length of lines at unity Pf.

The above equation represents loci of maximum power for different lengths of lines at unity power factor.

Considering a general load of $P_L + jQ_L$ connected to receiving end maintaining the sending end voltage constant. Then the receiving end current is given by

$$I_r = \frac{P_L - jQ_L}{V_r} \quad \text{------} (49)$$

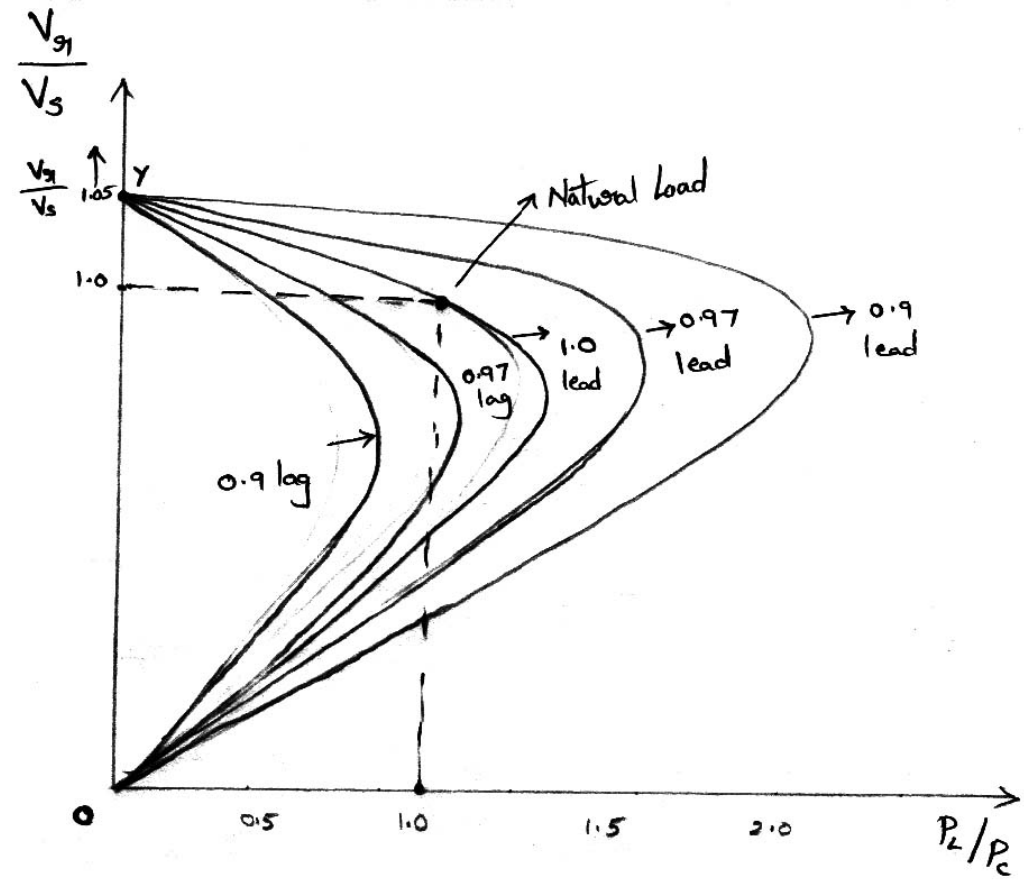
From equation

$V(x) = V_r \cos(l-x)\beta + j Z_0 I_r \sin\beta(l-x)$ and assuming the line to be loss less. Then we get,

$$V_s = V_r \cos\beta l + j Z_0 \frac{P_L - jQ_L}{V_r} \sin\beta l \quad \text{---} (50)$$

For a particular V_s and line length, this will be a quadratic equation in terms of V_r and consists of two roots. The below figure – 1 shows the relation between V_r/V_s as a function of normalized loading P_L/P_C.

As a function of normalized loading



From the figure we can conclude that,

1. For any loading there are two different values of V_r
2. There is a maximum power that can be transmitted for each load power factor.
3. Operating region of the power system is normally along the upper part of the curve where the receiving end voltage is 1.0 p.u.
4. As the maximum power print is reached, the system voltage is reduced much more than the increase in current and consequently there is a considerable reduction in power transmission. The voltage then finally collapses to zero and the system at the receiving end is considered to be effectively short circuited and so the power transmitted is Zero.
5. Point O corresponds to short circuit and point A corresponds to open circuit and in both cases the power transmitted is Zero.
6. The short length lines can be operated without any compensating devices but the long length lines cannot be operated without compensation because of Ferranti on of alternators during charging of the line and finally reduction of power transfer capability of the results in unstable operation of the power system.

COMPENSATED TRANSMISSION LINES

Compensation means to modify the electrical characteristics of a transmission line. The main objectives of compensation system is

1. To produce a flat voltage profile at all loading conditions.
2. To increase the maximum power transmission, which in turn improves system stability.
3. To generate reactive power according to the requirement economically while reducing supply side reactive burden.

When the surge impedance of the line is exactly equal to the actual load connected then a flat voltage profile is achieved.

If an effective surge impedance of the line is modified to have a virtual value Z_0^1, for which the corresponding virtual natural load V_0^2/Z_0^1 is equal to the actual load, then a flat voltage profile can be achieved.

The uncompensated surge impedance is $Z_0 = \sqrt{L/C}$

Which can be equal to $\sqrt{(X_l . X_c)}$

If the series or shunt reactances X_l and capacitance X_c are modified by compensation (connection of capacitors or reactors) to have a virtual surge impedance Z_0^1 and a virtual natural load P_0^1 for which

$$P_0' = \frac{V_0^2}{Z_0'} = P. \qquad \text{—— (51)}$$

Where

P is the actual power to be transmitted

V_0 is the rated voltage of the line

The compensation which performs a function of modifying Z_0 or P_0 is called as **surge impedance compensation** or **Z_0 – Compensation.**

The fundamental parameters of the transmission line Z_0 and θ influence transmission angle δ and in turn influences stability. This can be given by the below relation of power

$$P = \frac{V_s V_r}{Z_0 \sin\theta} \sin\delta \qquad \text{—— (52)}$$

To achieve a flat voltage profile by virtual surge impedance compensation, Z_0^1 is determined and in this connection the effective value of θ is reduced to improve stability.

There are two compensation strategies for this, first one is to apply series capacitors to reduce X_l and thereby to reduceθ, Since $\theta = \sqrt{(X_L/X_C)}$ at fundamental frequency. This is called as line **length compensation** or **θ - Compensation**.

Second one is to divide the entire line into shorter sections independent of one another which is called **compensation by sectioning**.

This is achieved by connecting constant voltage compensators in the sections along the line. The maximum transmissible power of shorter sections is less than the whole line and increase in maximum power which intun improves stability. All the above three compensations may be used together in a single transmission line.

Compensators: General classification

Generally the compensators are classified into passive and active compensators.

Passive Compensators :

Passive compensators constitute shunt reactors, shunt capacitors and series capacitors. Their principle of operation is to modify the natural inductance and capacitance. There are static devices. They can be permanently connected or switched according to the specification. There are used only for surge impedance compensation and line-length compensation. Shunt reactors are connected to compensate for the effects of distributed line capacitance which limits the voltage rise under open circuit or light loaded conditions.

Shunt capacitors are connected to generate reactive power which improves the voltage profile of the system. Series capacitors are connected for line length compensation. The effect of shunt and series capacitors on the transmission line is clearly illustrated in the preceding sections.

Active Compensators :

Active compensators are usually shunt connected devices which maintains a constant voltage profile at the terminals by generating or absorbing the required amount of reactive power in response to voltage variations at the point of their connection. These are controllable devices. Some examples of active compensators are synchronous condensers, thyristor switched capacitors (TSC) and thyristor controlled reactors. (TCR).

The surge impedance $Z_c = \sqrt{L/C} = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{X_L X_C}$

Let an inductance of L_{sh} be connected in shunt for compensation purpose and the net susceptance is given by,

$$j\omega c' = j\omega c + \frac{1}{j\omega L_{sh}} = j\omega c + \frac{j\omega c}{j\omega L_{sh}(j\omega c)}$$

$$= j\omega c - \frac{j\omega c}{\omega^2 c L_{sh}}$$

$$= j\omega c\left(1 - \frac{1}{\omega^2 c L_{sh}}\right)$$

$$= j\omega c\,(1-\alpha_{sh})$$

$$\text{where} \quad \alpha_{sh} = \frac{1}{\omega^2 c L_{sh}} = \frac{X_c}{X_{Lsh}}$$

And α_{sh} is known as the degree of shunt compensation due to compensation by the shunt inductance L_{sh}, the modified value of surge impedance is given by

$$Z_c' = \sqrt{\frac{j\omega L}{j\omega c(1-\alpha_{sh})}} = \frac{Z_c}{\sqrt{1-\alpha_{sh}}} \quad \left[\because Z_c = \sqrt{\frac{j\omega L}{j\omega c}}\right] \qquad \text{——— (53)}$$

Similarly, if a shunt capacitance is added, then the degree of shunt compensation α_{sh} is added, then the degree of shunt compensation α_{sh} is negative. Therefore we can conclude that compensating by shunt inductance increases the virtual surge impedance of the line and compensating by a shunt capacitance reduces the virtual surge impedance.

Let us consider the effect of series compensation on the surge impedance loading.

Consider a capacitance C_{se} per unit length which is placed in series for series compensation. Therefore the effective series reactance will be given by,

$$j\omega L - \frac{j}{\omega C_{se}} = j\omega L - \frac{j}{\omega C_{se}} \cdot \frac{j\omega L}{j\omega L}$$

$$= j\omega L\left[1 - \frac{1}{\omega^2 L C_{se}}\right]$$

$$= j\omega L\left[1 - \frac{X_{cse}}{X_L}\right]$$

$$= j\omega L\,(1-\alpha_{se}) \qquad \text{——— (54)}$$

Where $\alpha_{se} = X_{cse}/X_l$ which is known as the degree of series compensation.

The virtual surge impedance due to series compensation is given by Z_C^1,

$$Z_c' = \sqrt{\frac{j\omega L(1-\alpha_{se})}{j\omega C}} = Z_c\sqrt{(1-\alpha_{se})} \quad \text{—(55)}$$

Taking both series and shunt compensation cases simultaneously, we get

$$Z_o' = \sqrt{\frac{j\omega L'}{j\omega C}} = Z_c\sqrt{\frac{1-\alpha_{se}}{1-\alpha_{sh}}} \quad \text{—— (56)}$$

Therefore the virtual surge impedance loading is given by

$$P_c' = P_c\sqrt{\frac{1-\alpha_{sh}}{1-\alpha_{se}}} \quad \text{—— (57)}$$

Now, the wave number β is also modified to β^1 and can be written in terms of degree of series and degree of shunt compensations as,

$$\beta' = \beta\sqrt{(1-\alpha_{se})(1-\alpha_{sh})} \quad \text{———— (58)}$$

From equations (56) and (57) it is evident that

1. For a constant degree of series compensation, capacitive shunt compensation decreases the virtual surge impedance loading of the line.
2. Inductive shunt compensation increases the virtual surge impedance.
3. Inductive shunt compensation decreases the virtual surge impedance loading of the line.

If the inductive shunt compensation is 100% the virtual surge impedance becomes infinite (from equation (56)) and the surge impedance loading of the line becomes zero (from equation (57)). As we know that a flat voltage profile exists at a load which is equal to surge impedance loading. Therefore, if inductive shunt compensation is 100% a flat voltage profile exists at zero load, where Ferranti effect comes into picture at zero loaded conditions of a line which can be eliminated by connecting shunt reactors. Under heavy loading conditions, a flat voltage profile can be obtained by using shunt capacitances.

For example, if we desire a flat voltage profile corresponding to 1.25 Pc without any series compensation, the shunt capacitance compensation required can be calculated by equation (57)

From equation (57) where P_C^1 corresponds to 1.25 P & α_{se}=0

We get,

$$1.25 = \sqrt{\frac{1-\alpha_{sh}}{1-0}} \quad [\because \alpha_{se} = 0]$$

$$(1.25)^2 = 1-\alpha_{sh} \Rightarrow 1.5625 = 1-\alpha_{sh}$$

$$\alpha_{sh} = -0.5625 \text{ p.u.}$$

Similarly a flat voltage profile can be obtained for heavy loading conditions by series compensation also. From equation (57) assuming degree of shunt compensation α_{sh} as Zero, we get the series compensation required for a particular loading.

For example, if we desire a flat voltage profile corresponding to 1.25 P_C without any shunt compensation the series capacitance compensation required can be calculated by the equation (57).

From equation (57) where P_C^1 corresponds to 1.25 P and

$\alpha_{sh} = 0$, we get

$$1.25 = \sqrt{\frac{1-0}{1-\alpha_{se}}}$$

$$1.5625 = \frac{1}{1-\alpha_{se}}$$

$$1.5625 - 1.5625\,\alpha_{se} = 1$$

$$1.5625\,\alpha_{se} = 0.5625$$

$$\therefore \alpha_{se} = 0.36 \text{ p.u.}$$

Generally, the voltage control of the system using series capacitors is not preferred because of the lumped nature of series capacitors and normally used for improving stability of the system.

Distributed compensation

The effect of distributed compensation on line charging reactive power is discussed below.

At no load condition, the reactive power generated in the line by virtue of line charging (Q_s) has to be absorbed by the synchronous machines connected to the terminals. And Q_S is given by rewriting the equation (34) from uncompensated lines, i.e. the reactive power supplied by the synchronous generator for a symmetrical radial line.

$$Q_s = -P_c' \operatorname{Tan}(\beta l)' \qquad \text{——— (59)}$$

Where βl is the electrical length in radians.

For a compensated symmetrical Radial line, the above equation can be modified as,

$$Q_s = -P_c' \operatorname{Tan}\left(\frac{\beta l}{2}\right)' \qquad \text{——— (60)}$$

By virtue of substantial degree of series capacitive compensation or shunt inductive compensation, the electrical length of the line tends to be considerable reduced enough and therefore we can write

$$\operatorname{Tan}(\beta l)' = (\beta l)'$$

$$\text{and} \qquad \operatorname{Tan}\left(\frac{\beta l}{2}\right)' = \left(\frac{\beta l}{2}\right)'$$

Substituting the values of P_C^1 and $(\beta l)^1$ from previous equations (57) and (58) in the equations (59) and (60), for a compensated symmetric radial line we get,

$$Q_s' = -P_c' \, Tan\left(\frac{\beta l}{2}\right)'$$

$$= -P_c'\left(\frac{\beta l}{2}\right)'$$

$$= -P_c\sqrt{\frac{(1-\alpha_{sh})}{(1-\alpha_{se})}}\;\frac{\beta l}{2}\sqrt{(1-\alpha_{se})(1-\alpha_{sh})}$$

$$\because \; P_c' = P_c\sqrt{\frac{1-\alpha_{sh}}{1-\alpha_{se}}} \rightarrow (57)$$

and

$$\because \; \beta' = \beta\sqrt{(1-\alpha_{se})(1-\alpha_{sh})} \rightarrow (58)$$

$$= \frac{-P_c\,\beta l}{2}(1-\alpha_{sh})$$

$$\therefore \boxed{Q_s' = -P_c\,\frac{\beta l}{2}(1-\alpha_{sh})} \quad \text{——— (61)}$$

From equation (61) we can conclude that :

1. The equation is independent of the degree of series compensation (α_{se}). Therefore it is clear that the series compensation has no effect on the line charging reactive power at no load, which has to be supplied by synchronous generators connected at the terminals.
2. If $\alpha_{sh} = 0$ which implies that there is no shunt inductive compensation, the series compensated line generates approximately as much line charging reactive power at no load as by a completely uncompensated line of the same length.
3. If the line is long enough, then the reactive power demand at no load on the terminal synchronous generators rises and consequently the line requires series compensation as far as stability is concerned, since the synchronous generators cannot absorb the excessive reactive power without sacrificing their stability.
4. The optimal solution for this problem is to have additional inductive shunt compensation along with the series compensation.

Consider a shunt compensation which is uniformly distributed along the line and regulated which is similar to compensation by sectioning.

Let the line be operating at natural loading condition,

From equation (41) we can have,

$$\frac{P}{\sin\delta} = \frac{P_c}{\sin\beta l} \qquad \left(\text{if } P = P_c \text{ and } \delta = \beta l\right)$$

Which implies the transmission angle δ equals to the electrical length of the line. Since there is no series compensation assumed (α_{se}=0) shunt compensation could be regulated continuously such that P_C^1 = P and $\beta^1 l$ =δ at all conditions.

Now the previous equation changes to,

$$\frac{P}{\delta} = \frac{P_c'}{\beta' l}$$

Substituting the values of P_C^1 and β^1 from the equations (57) and (58) we get

$$\frac{P}{\delta} = P_c\sqrt{\frac{1-\alpha_{sh}}{1-\alpha_{se}}} \bigg/ \beta\sqrt{(1-\alpha_{se})(1-\alpha_{sh})} \cdot l$$

Since degree of series compensation $\alpha_{se} = 0$

$$\frac{P}{\delta} = \frac{P_c\sqrt{(1-\alpha_{sh})}}{\beta l\sqrt{(1-\alpha_{sh})}}$$

$$\frac{P}{\delta} = \frac{P_c}{\beta l}, \quad \text{which is a Constant} \quad \text{---(62)}$$

Since $P_c/\beta l$ is a Constant the above equation (62) shows a liner relation between P and δ and the slope is given by,

$$\frac{P_c}{\beta l} = \frac{V_0^2}{Z_0\beta l} \qquad \because P_c = \frac{V_0^2}{Z_0}$$

$$\frac{P_c}{\beta l} = \frac{V_0^2}{\sqrt{L/C}\cdot\omega\sqrt{LC}\cdot l} = \frac{V_0^2}{X_l} \qquad \left[\begin{array}{l}\because Z_0 = \sqrt{L/C}\\ \because \beta = \omega\sqrt{LC}\end{array}\right]$$

The power angle relation is given below.

$$P = \frac{V_0^2}{X_l}\sin\delta.$$

The slope at $\delta = 0$ is $\frac{V_0^2}{X_l}$ and the P-δ straight line is tangent to P-δ characteristic

Of the totally shunt compensated line as show in the figure (4). The line with 100% shunt compensation behaves exactly as a series inductance.

Figure – A illustrates that if shunt compensation could be adjusted continuously, the power transmitted could be infinite.

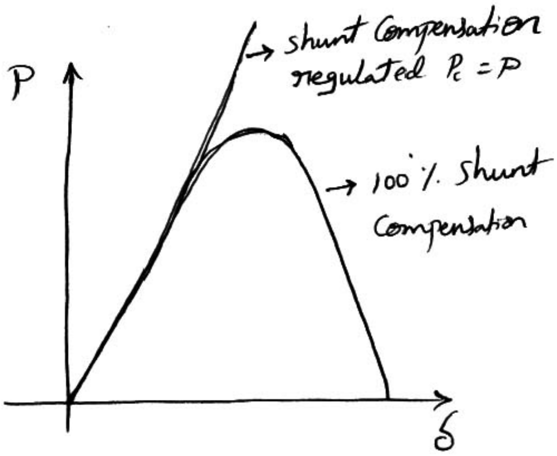


Fig-A

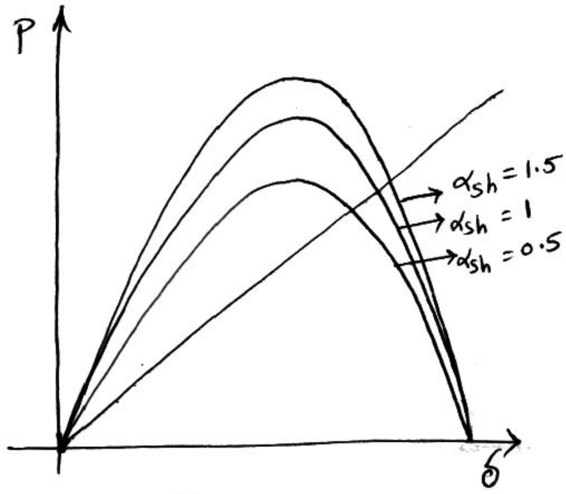


Fig-B

Fig A – Effect of regulated and distributed compensation

Fig B – P - δ curves for different α_{sh}.

Figure – B shows power angle characteristics for different values of shunt compensations α_{sh}, where the peak value of the power is given by

$$P'_{max} = \frac{V_0^2}{Z_0 \sin \beta' l}$$

Finally, we can conclude that as power varies, the shunt compensation should also be varied in order to have a straight line characteristic operation.

If P is varied faster as compared to the shunt compensation the system will operate along P-δ current characteristic and the system will go to unstable for angle ($\delta > \prod/2$). When $P > P_C^1$ the shunt compensator required is capacitive.

The shunt compensation reactors increase the virtual surge impedance of the line and hence decrease the surge impedance loading. With $\alpha_{sh} = 1$ i.e., the shunt compensation is 100% the voltage profile is flat at no load. Shunt compensating reactors cannot be distributed uniformly along the line but normally connected at the end or at the mid point of the line at any intermediate substation.

If switched shunt capacitors are used, they should be disconnected under light loaded condition, otherwise they may lead to Ferro resonance when transformers are present.

The basic idea of series compensation is to directly cancel the inductive reactance of the line, increase maximum power transmission, reduction in transmission angle and increase in surge impedance loading. Practically it is not desirable to exceed series compensation beyond 80%. If the line is 100% compensated, the line behaves as a purely resistive element and would result into series resonance. The location of series capacitors is also governed by economical factors and fault currents. And the rating will depend on the maximum fault current likely to flow through capacitor.

SERIES COMPENSATION

Capacitors are also can be treated as chief sources of reactive power available at the load end or receiving end. They are again classified into two types by the virtue of their mode of connection.

a) Series capacitors

b) Shunt capacitors

The main objective of capacitor placement whether they are connected in series or parallel is to compensate reactive power and to consequently improve power factor and voltage profile of the system.

The series capacitors (capacitors which are connected in series with the lines) directly neutralize the inductive reactance of the system to which it is connected. Since the effect of series capacitor can be considered as a negative reactance in series, the net impedance will be $Z' = R + j\,(X_L - X_c)$. And therefore the voltage drop IZ' is reduced. Application of a series capacitor to a feeder and its vector diagram representation is shown in figure-4(a) and figure-4(b)

Without series capacitor **With Series capacitor**

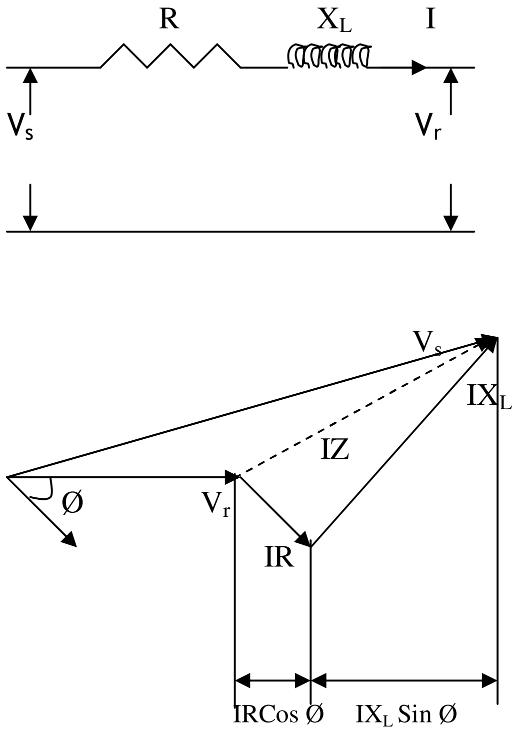


Fig – 4(a) Without series capacitor

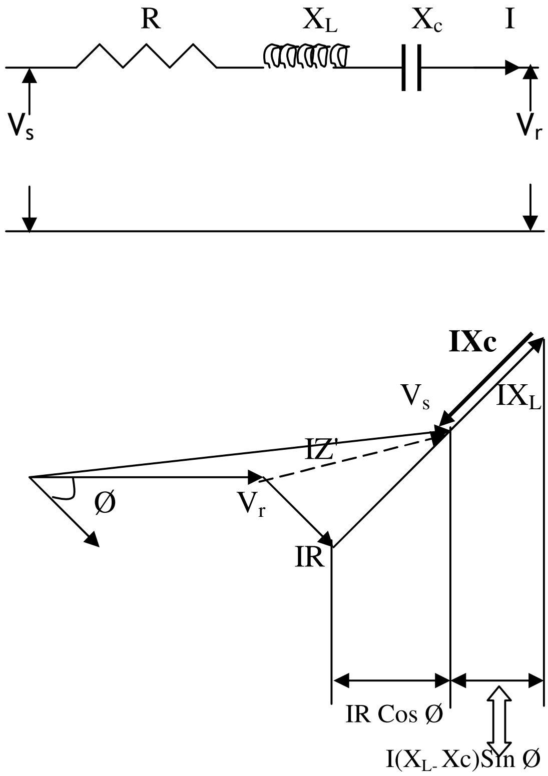


Fig – 4(b) With series capacitor

Voltage drop of the feeder without series capacitor is expressed as

$$V_D = I\,R\,Cos\,Ø \;+\; I\,X_L\,Sin\,Ø$$

Where, R = Resistance of the feeder

X_L= Inductive reactance of the feeder

Ø = angle between the receiving end voltage V_r and current I in the feeder

Resultant Voltage drop of the feeder with the application of series capacitor is

$$V'_D = I\,R\,Cos\,Ø + I(X_L - Xc)\,Sin\,Ø$$

Where, Xc = Capacitive reactance of the series capacitor

Since after placing a capacitor in series with a feeder the resultant impedance will be

$$Z' = R + j\,(X_L - X_c).$$

Therefore I X_c is the reduction in the voltage drop after series compensation.

- If $X_L = X_c$ then $V'_D = I * R$ i.e., the line has only resistive drop owing to the resistance of the line and zero inductive drop.

- If Cos Ø = 1 then Sin Ø = 0 and therefore, $I(X_L - Xc)$ Sin Ø = 0 and $V'_D = I\,R$. In such unity power factor applications, series capacitors have practically no value.
- To decrease the voltage drop considerably between the sending end and the receiving end by applying a series capacitor then the power factor should be lagging.
- For long transmission lines where the net reactance is high, series capacitors are effective for enhancing system stability

SHUNT COMPENSATION

The shunt capacitors (capacitors connected in parallel with the lines) are widely used in distribution systems for compensating reactive power and to improve power factor. Shunt capacitors draw a leading current which counteracts lagging component of the inductive load current (some or the entire part) at the point of installation. Thus it modifies the characteristics of inductive load by drawing leading current. A shunt capacitor has a similar effect as an overexcited synchronous generator or a motor. Application of a shunt capacitor to a feeder and its vector diagram representation is shown in figure-5(b) and figure-5(b)

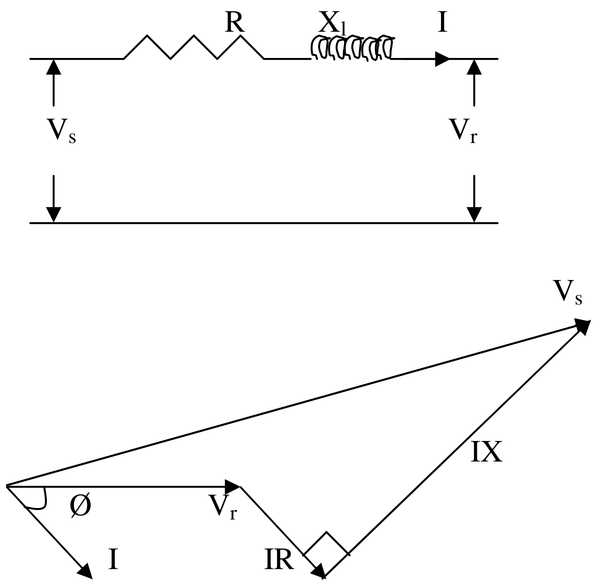


Fig – 5(a) Without shunt capacitor

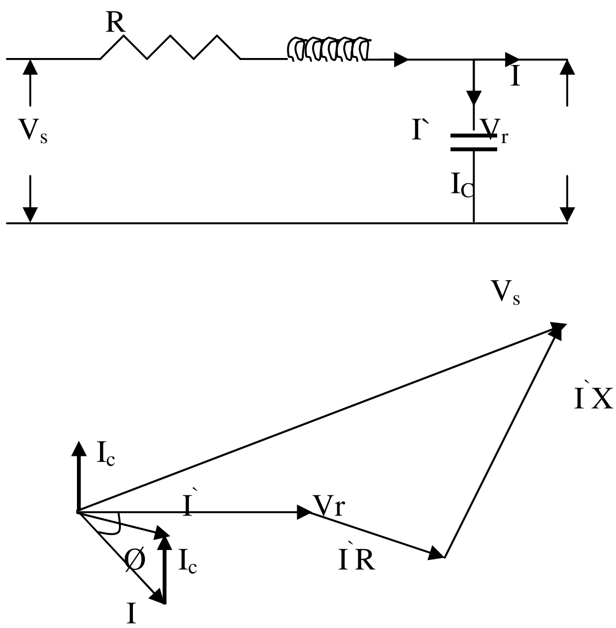


Fig – 5(b) With shunt capacitor

Voltage drop of the feeder without shunt capacitor is expressed as

$$V_D = I_R R + I_X X_L$$

Where, R = Resistance of the feeder

X_L= Inductive reactance of the feeder

I_R = real power component of the current (in phase)

I_X = reactive component of current (out of phase, lagging the voltage by 90^0)

Resultant Voltage drop of the feeder with the application of shunt capacitor is

$$V'_D = I_R R + I_X X_L - I_C X_L$$

Where, Ic = reactive component of current leading the voltage by 90^0 (out of phase)

The voltage rise due to the installation of shunt capacitor can be expressed from the difference between the above two expressions

$$V_R = I_C X_L \quad \text{Volts.}$$

POWER FACTOR CORRECTION

Loads on electric power systems include two components like real power and reactive power. Real power is exclusively generated at generating stations and reactive power can be generated either by generating plants or at the load side through capacitors. When the reactive power is supplied by power plants then the size of each system component like generators, transformers, transmission and distribution lines, and protective equipment will be greater than before and it is not economically feasible. Application of shunt capacitors can diminish all these conditions by decreasing reactive power demand on generators. There are released generation, transmission and distribution substation capacities, reduction in losses and loadings in lines and distribution transformers owing to reduction in line currents from capacitor locations. Shunt capacitors are the most economical sources to meet the reactive power requirements of inductive loads and transmission lines operating at lagging power factor.

Assume that a load is supplied with a real power P, lagging reactive power Q_1, and apparent power S_1 at a lagging power factor of

$$\cos \text{Ø}_1 = P / S_1$$

$$\cos \text{Ø}_1 = P / (P^2 + Q_1^2)^{1/2}$$

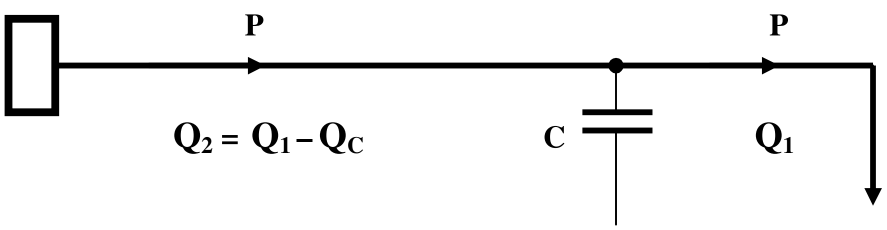


FIG – 6 Illustration of power factor correction

Q_C kVAR is installed across the load, then the reactive power of the load is compensated and the net reactive power will be $Q_2 = Q_1 - Q_C$ and consequently power factor is corrected (improved) from Cos $Ø_1$ to Cos $Ø_2$.

The new corrected power factor is expressed as,

$$\text{Cos } Ø_2 = P / S_2$$

$$\text{Cos } Ø_2 = P / (P^2 + Q_2^2)^{1/2}$$

$$\text{Cos } Ø_2 = P / [P^2 + (Q_1 - Q_C)^2]^{½}$$

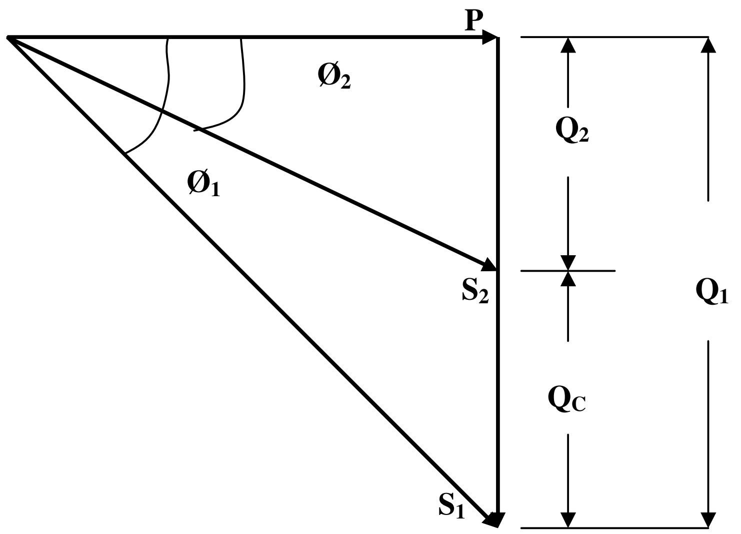


Fig-7 Illustration of power factor correction

Therefore we can conclude that by connecting a capacitor across the load and supplying a reactive power of Q_c, the apparent power is decreased from S_1 kVA to S_2 kVA and the reactive power is decreased from Q_1 kVAR to Q_2 kVAR

NUMERICAL EXAMPLE

Example 3

Assume a 4 kV single phase circuit which feeds a load of 450 kW and operates at a lagging power factor of 0.75. If it is desired to improve the power factor, determine the following.

a) The reactive power consumption.

b) The new corrected power factor after installing a shunt capacitor bank with a rating of 400 kVAR

Solution:

Before the installing shunt capacitors for power factor correction,

The current drawn by the load can be calculated by

$P = V\ I\ \text{Cos}\ Ø_1$

$450 = 4 * I * 0.70$

$I = 450 / 4 * 0.70$

$I = 160.714$ A

Apparent power $= S_1 = V * I$

$= 4 * 160.714$

$= 642.85$ kVA

a) Reactive power consumption can be calculated by

$Q_1 = S_1 * \text{Sin}\ Ø_1$ [$\text{Cos}\ Ø_1 = 0.70$, therefore $Ø_1 = 45.57^0$]

$= 642.85 * \text{Sin}\ (45.57^0)$

$\cong 459$ kVAR

After power factor correction by installing 450 kVAR Capacitor bank

$Q_2 = Q_1 - Q_C$

$= 459 – 400$

$= 59$ kVAR

b) Therefore the new corrected power factor can be calculated by

$\text{Cos}\ Ø_2 = P / [\ P^2 + (Q_1 - Q_C)^2\ ]^{1/2}$

$= 450 / [\ 450^2 + 59^2\ ]^{½}$

$= 450 / 453.85$

$= 0.9915$ or 99.15 percent.

ADVANTAGES AND DISADVANTAGES OF DIFFERENT TYPES OF COMPENSATION EQUIPMENT FOR TRANSMISSION SYSTEMS

S.NO	COMPENSATING EQUIPMENT TYPE	ADVANTAGES	DISADVANTAGES
1	Synchronous Generator	Offers flexibility for all load conditions Smooth variation of generation of reactive power	Size of the machine increases and this method is not economically feasible.
2	Shunt Reactor (switched or fixed)	Simple in construction and principle of working	Fixed value of degree of series compensation
3	Shunt Capacitor (switched or fixed)	Simple in construction and principle of working	Fixed value of degree of shunt compensation Switching transients may occur Step by step variation of generation of reactive power

4	Series Capacitor	Simple in principle of working Performance is insensitive to location when compared to shunt compensation	Fixed value of degree of series compensation Limited overload capacity Requires over voltage protection and sub harmonic filters Installation is relatively not easy
5	Synchronous Condenser	Completely controllable Flexible in compensation Good over load capacity Induces less harmonics	High maintenance required Slow response to control Performance sensitive to location Requires strong foundations
6	Thyristor controlled reactor (TCR)	Very fast response Completely controllable Flexible in compensation Can be rapidly repaired after failures. No effect on fault level	Generates harmonics Performance sensitive to location
7	Thyristor switched capacitor (TSC)	Can be rapidly repaired after failures. Induces no harmonics into the system	No absorbing capability to limit over voltages Complex controls Problem of low frequency resonance with the system Performance sensitive to location
8	Polyphase saturated reactor	Rugged construction Good overload capability No effect on fault level	Noisy in operation Performance sensitive to location

		Can be rapidly repaired after failures. Induces less harmonics into the system	Essentially fixed in value of compensation

VOLTAGE CONTROL

INTRODUCTION

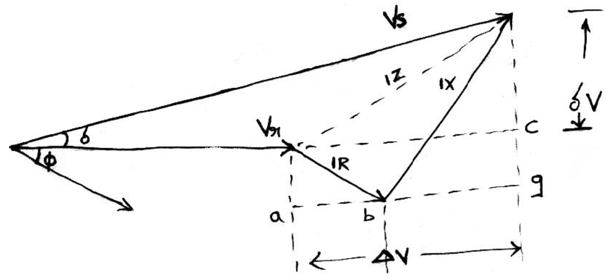
For satisfactory operation of the general power system loads like lamps, motors etc., it is essential that the voltage at the consumer terminals should be maintained constant (or) within prescribed limits ($\pm$ 6% of supply voltage). Large variations in the voltage cause inefficient operation and malfunctioning of several consumer appliances. Even the maximum power transfer for long lines is limited by the magnitude of voltages at both sending and receiving ends, reactance between both ends and the sine of angle between the two voltages. The primary reason for voltage variation is the variation in the connected load on the system. With the increase in the connected load on the supply system the voltage falls since there is an increase in the voltage drop due to voltage drop in the alternator synchronous impedance, transmission line transformer impedance, feeders and distributors. If the connected load suddenly falls, then the voltage also rises.

In order to maintain the voltage within prescribed limits, voltage control equipment should be placed at suitable locations such as generating stations, transmission substations, intermediate switching stations, distribution substations supplying to feeders etc., since the transmission and distribution network is a geographically extensive one. The voltage control equipment is located or provided at more than one point in the power system.

In a DC system, the DC voltage can be controlled very easily by employing over – compound generators in case of feeders of equal lengths. For different length feeders and unequal voltages series wound DC Generators also called as feeder boosters are used.

EFFECT OF REACTIVE POWER ON VOLTAGE

Consider a transmission network comprising of a resistance R, reactance X, impedance Z, sending end voltage (V_s) and receiving voltage (V_r) and power angle (δ). Let us derive the relation for reactive power and the system voltage from the below phasor diagram.



From the phasor diagram, we can write that,

$$V_s^2 = (V_r + \Delta V)^2 + \delta V^2 \quad \text{——— (1)}$$

$$V_s^2 = (V_r + IR\cos\phi + IX\sin\phi)^2 + (IX\cos\phi - IR\sin\phi)^2$$

As it is evident that $P = V_r I\cos\phi \Rightarrow I\cos\phi = \frac{P}{V_r}$

$Q = V_r I\sin\phi \Rightarrow I\sin\phi = \frac{Q}{V_r}$

Rearranging according to the above two expressions of active and reactive powers, we get

$$V_s^2 = \left[V_r + \frac{RP}{V_r} + \frac{XQ}{V_r}\right]^2 + \left[\frac{XP}{V_r} - \frac{RQ}{V_r}\right]^2$$

$$V_s^2 = \left[V_r + \frac{RP + XQ}{V_r}\right]^2 + \left[\frac{XP - RQ}{V_r}\right]^2 \quad \text{——— (2)}$$

Comparing the two equations (1) and (2) we get,

$$\Delta V = \frac{RP + QX}{V_r} \quad \text{and} \quad \delta V = \frac{XP - RQ}{V_r}$$

Assuming $\delta V << (V + \Delta V)$ then the equation can be modified to,

$$V_s^2 = \left[V_r + \frac{RP + XQ}{V_r}\right]^2$$

$$V_s = V_r + \frac{RP + XQ}{V_r}$$

$$V_s - V_r = \frac{RP + XQ}{V_r} = \Delta V$$

The arithmetic difference between the sending end voltage V_s and receiving end voltage V_r is approximately given by $(RP + XQ) / V_r$

Assuming the line to be a loss less line, where R = 0

$$\boxed{V_s - V_r = \frac{XQ}{V_r}} \quad (or) \quad \boxed{Q = \frac{V_r (V_s - V_r)}{X}}$$

From the above relations we can conclude that the reactive power flow depends primarily on the arithmetic difference between the sending and receiving end magnitudes and it flows from higher voltage end to the lower voltage end.

METHODS OF VOLTAGE CONTROL

In an AC supply system, the voltage can be controlled by the following methods.

1. Excitation control through voltage regulators at generating stations.
2. Employing Tap changing transformers at both sending and receiving end of the transmission lines, industries, substations (both distribution x Transmission).
3. Employing Booster Transformers.
4. Use of Induction regulators.
5. Inserting series capacitors in long EHVAC transmission lines.
6. Employing Switched or Fixed shunt Capacitor banks.
7. Switching in shunt reactors during light loads.
8. Use of synchronous condensers and thyristorised control for step less control of reactive power and voltage.

EXCITATION CONTROL

The terminal voltage of the alternator varies when the load on the supply system changes which is due to voltage drop in the synchronous impedance of the alternator. The voltage of the alternator cannot be controlled by adjustment of speed, as they have to be run at a constant speed. So the voltage of the alternator can be controlled by using excitation regulation which can be regulated by use of automatic or hand regulator acting in the field circuit of alternator exciter.

The quick acting voltage regulators based on the over-shooting the mark principle gives the quick response which cannot be obtained either by variation of field circuit resistance or change in exciter voltage due to high inductance of alternator. There are two types of automatic voltage regulators. They are
1. Tirril regulator and
2. Brown – Boveri regulator

1. Tirril Regulator: A vibrating type voltage regulator.

The control can be possible by rapidly opening and closing a shunt circuit across the exciter rheostat. In which a fixed resistance is cut in and cut out of the exciter field circuit of the alternator.

2. Brown – Boveri Automatic voltage Regulator:

It works on the principle of the "Over shooting the mark" but differs from Tirril Regulator. In Tirril regulator the resistance is first completely inserted them completely cut out whereas in this type, the regulating resistance is gradually varied in small steps. The system remains rest under steady conditions and the maintenance is less when compared to Tirril regulator.

The excitation control method is limited to small isolated systems because the system becomes unstable if the excitation is below a certain level and also it causes over heating of the rotor if the excitation is above certain level.

TAP CHANGING TRANSFORMERS

As the excitation control method is limited only to small isolated systems, we have to go for other methods for long transmission lines. One method among them is a Tap changing transformer one which is usually employed where main transformer is necessary

The basic principle is based on changing the ratio of transformation, which can be obtained by adjusting the turns on the primary or secondary depending upon the requirement.

Principal tapping is one in which the tapping on the hr winding when connected to rated voltage gives rise to rated voltage on the LV side.

Positive tapping, is tapping in which the number of turns are more than that of principal tapping whereas a negative tapping is one in which the tapping have loss number of turns than the principal tapping.

Most of the transformers carry rated KVA on over voltage tap and rated current on a reduced voltage tap.

Generally tapping is provided on high voltage side due to following reasons.

1. Smooth and fine control output voltage can be possible as the number of turns on HV side is more.
2. Owing to insulation constraints the LV winding is placed nearer to the core and therefore it is difficult to tap LV winding.
3. Tap chargers on HV side has to carry low currents though it will need more insulation.

Generally there are two types of tap changing's

1. OFF load tap changing.
2. ON load tap changing.

1. <u>OFF load tap changing :</u>

In this method, there is a need to switch off the supply to the transformer whenever the tap changing takes place. This can be carried out by manually operated switches. This is a very economic method of changing the turn – ratio of a transformer. This is used for occasional adjustments in distribution transformers which are provided with $\pm$ 5% and $\pm$ 2% taps.

2. ON Load Tap Changing :

Almost all the power transformers of large ratings use this type of tap changing. This is based on the principle that, the tap changing on the transformer takes place while delivering the load. The operation can be possible either by local or remote control and also a handle is filled for manual operation in case of emergency if necessary.

The main features of an on-load tap changer is that there is no need to open the main circuit whenever sparking takes place and also no part of the tapping should get short circuited. They are provided with an impedance to limit the short circuit current during the operation.

In general they are also called as resistor or reactor type on-load tap-changers, because in place of impedance they use either resister or centre – tapped reactor. Now days, they are designed by a pair of resistors, which invariably limits the current.

BOOSTER TRANSFORMER

If the voltage of a feeder has to be controlled at a point far away from the main transformer and if there is no provision for a tap changing gear in the main transformer then we have to use a special transformer which is known as Booster transformer. The primary of booster transformer is supplied from the secondary of the regulating transformer which is fitted with on load tap changing gear. The regulating transformer output is connected to the primary of the booster transformer in such a way that the voltage injected in the line is in phase with the supply voltage. The system becomes expensive if the regulation is required at a point where a main transformer is to be placed and it also requires more floor space and increases the losses.

INDUCTION VOLTAGE REGULATORS

Induction voltage regulator works on the idea of rotating the primary with respect to secondary. In an Induction voltage regulator the secondary voltage can be varied from zero to maximum value by adjusting the position of the primary - coil axis with respect to the secondary – coil axis, which in turn depends upon the ratio of turns in the two windings. This is also known as step – down transformer. Generally, in this regulator the secondary is connected in series with the circuit and the primary is connected across the circuit to be regulated. They may of single-phase or three-phase and consists of stator and rotor.

The main advantages of this type of voltage control are:

1. They have simple and rugged construction.
2. It gives reliable operation
3. The load and power factor variations do not effect its operation.
4. It provides smooth voltage variation without any arcing or short – circuiting of turns.

The main disadvantage is that it is more expensive than the transformers with tap-changing mechanism. Also they have small magnetizing currents.

The most important application of induction voltage regulator is in distribution systems to maintain the load voltage at a constant value under all load conditions.

SHUNT REACTORS

Shunt Reactors are generally used to control steady state over voltages when operating under light-load conditions. The reactive power generated by the capacitance causes high voltages if the shunt reactors were not employed. The shunt reactive compensation is kept permanently in order to avoid over voltages and insulation stresses followed by sudden load rejection. The shunt reactors reduce the power transfer capability of the line. Generally they appear like power transformers. They are connected to the low – voltage tertiary winding of a transformer through a suitable circuit breaker. Generally, oil immersed magnetically shielded reactors with gapped core are employed.

STATIC SHUNT COMPENSATION

Due to recent advances in power electronics and their component ratings these compensating techniques are provided to be far superior and have a step less control of variable compensation. A thyristorised control of shunt reactors and capacitors is provided. The stability improvement and transient voltage control can be possible by using static VAR system (SVS).

The thyristors in capacitor control are made to conduct for long time during peak load period and the thyristors in reactor circuit are made to conduct for long time during light-load period. Therefore step less variation of shunt compensation can be obtained by using static compensation.

SYNCHRONOUS CONDENSERS

Generally synchronous condensers are specially designed synchronous motors, which are used to control receiving end voltage of a transmission line. According to the load on the transmission line, by varying its excitation the watt less kVA is automatically varied. Due to low losses the efficiencies of these machines are high and hence they draw less current. The phase angle between applied voltage and current is 90^0. The main advantages of synchronous condensers are:

1. Both ends of transmission line can be maintained with same voltage.
2. At heavy loads power factor can be improved.
3. As high terminal reactances are used better protection is possible to the line.

The main disadvantages are supply interruption increases if synchronous condenser comes out of synchronism and also short-circuit current increases.

The effect of series and shunt capacitors has been discussed earlier in this chapter.

MALLA REDDY ENGINEERING COLLEGE (AUTONOMOUS)

IV B.Tech I semester End Examination

Department of Electrical & Electronics Engineering

MODEL QUESTION PAPER

Subject: Power System Operation and Control (PSOC) **Max. Marks: 60**

Duration: 3 Hours **Marks: 60**

PART-A

Answer All Questions **5X2 =10Marks**

1. What are the assumptions of B –Coefficients?
2. Define Spinning Reserve?
3. Define Speed Governor
4. What Is Control area concept?
5. What are the disadvantages of Series Compensation?

PART B **5X10=50Marks**

Answer any 5 out of 8 Questions

1. Derive and expression for Transmission loss and explain the Significance of penalty factor?

2. Explain about Hydro Thermal co-Ordination with necessary equations?

3. Draw the complete block diagram for Single area load frequency control system and explain in detail about Steady State analysis for controlled case with necessary equation?

4. Deduce the expression for static frequency and tie line power in an identical two area system.

CASE STUDY; A Study Of Power System Security And Contingency Analysis

An important factor in the operation of a power system is the desire to maintain system security. System security involves practices suitably designed to keep the system operating when the components fail. An operationally “secure” power system is the one with low probability of system blackout or equipment damage. A secured system is one which has the ability to undergo a set of disturbances without getting into an emergency condition. In other word a normal operating condition of a power system is considered to be secured if there is neither any occurrence of over loading of any neither equipment nor transient instability due to a set of credible contingency. The secured condition satisfies not only the loading and operating constraints but a third set of constraints known as the security constraints. In the modern days the power system is becoming wide and complex. Contingency Analysis (CA) is critical in power system analysis. This work reviews the techniques for contingency analysis based on line flow analysis. Contingency analysis is performed considering the line and generator outage contingencies, in order to identify the effect